# Roll angle of autocollimator measurement method based on hollow cube corner reflector* 

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#### Abstract

In this paper, a method of measuring roll angle with hollow cube corner reflector (HCCR) is proposed. Firstly, the space coordinate vector relationship between the mirror and the autocollimator is established by using the Euler rotation relationship, and the structure of the HCCR is designed. Secondly, through the actual measurement of the HCCR reflector's angle sensitivity, the specific formula of roll angle measurement is obtained. The experimental results show that the method can be used to measure the roll angle, and the maximum root mean square ( $R M S$ ) error is $1^{\prime \prime}$ in the range of 350 ", and the repeatability of the experiment is better than $2.7^{\prime \prime}$. On the basis of retaining the traditional autocollimator, the plane mirror is replaced with HCCR, which realizes the measurement of the roll angle of the autocollimator. This method does not change the internal structure of the autocollimator system, and there is no process error caused by special processing technology, so as to retain the autocollimator's own long measurement distance, high measurement accuracy, high system stability, and large range of measurement performance, and it can also be used in industrial production.


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Angle measurement system plays an important role in precision machining, machine tool alignment, precision instrument calibration and other fields ${ }^{[1,2]}$. At present, the research on the new angle measurement method based on the optical principle is becoming a research hotspot in the scientific community and has received extensive attention due to its higher measurement stability and accuracy ${ }^{[3]}$.

Optical angle measurement methods mainly include polarization detection method ${ }^{[4,5]}$, diffraction grating method ${ }^{[6]}$, heterodyne interferometry ${ }^{[7,8]}$ and autocollimation method ${ }^{[9]}$, etc. However, for the simultaneous measurement of three-dimensional space attitude angles (pitch angle, yaw angle, roll angle), there is still a lack of effective solutions. The main reason for this situation is the technical difficulty of measuring the roll angle ${ }^{[10,11]}$.

Currently, several new methods of roll angle measurement have been reported. In 2019, QI et al ${ }^{[12]}$ realized the measurement of roll angle by means of polarization change and heterodyne interferomete. By using two acou-stro-optic modulators to generate two beams with different frequencies but the same polarization state, and then combining the two beams into dual-frequency light, the hete-
rodyne signal was significantly enhanced, so that the roll angle could be measured according to the gain coefficient of the heterodyne signal. Although this method can measure the roll, it not only needs to use the photoacoustic modulator to obtain the two light sources with the same polarization and different frequency, but also needs to adjust the angle position of the quarter wave plate to realize the overlap of the two beams. This method is complicated to the beam processing, and the adjustment of the quarter wave plate position to achieve the overlap of two beams is easy to lead to the system in the process of measurement, poor stability, complex operation and other defects. In the same year, CAI et al ${ }^{[13]}$ used a laser diode as a laser light source, and used two photodetectors to detect the relative displacement of two parallel laser light sources. The method is simple, and the measurement accuracy of the roll angle within the measurement range of 1 m is $\pm 1.2^{\prime \prime}$. However, the laser light source has the problem of angular drift of the laser beam for roll angle measurement, which results in the non-parallelism of the measurement beam, which will seriously affect the measurement distance. Then, in 2020, REN et al ${ }^{[14]}$ proposed an improved parallel

[^0]light measurement technique, using a grating and a mirror to generate a pair of parallel beams, and using a photodetector (PD) to record the change of spot displacement, so as to obtain the relationship between the roll angle and the measurement system, achieving a resolution of $0.5^{\prime \prime}$ and a measurement accuracy of $1.1^{\prime \prime}$. The method uses PD to measure the spot displacement information to realize the roll angle, and has a simple structure and high resolution and measurement accuracy. However, as the working distance increases, the measurement performance of the PD will be greatly reduced, so it's not suitable for large range measurement. In 2021, we proposed a modified cube corner reflector (MCCR) structure with a large range of three-dimensional measurement ${ }^{[15]}$. It is obtained by changing the effective angle of the three reflector mirrors of the cube corner reflector to be non- $90^{\circ}$. This structure achieves a large range of three-dimensional angle measurement, the measurement accuracy of the roll angle is $40.43^{\prime \prime}$ within the measurement range of $10^{\circ}$. However, this structure requires the derivation of the second reflection mirror from one reflection mirror, so it can only be obtained by cutting the standard glass cube corner mirror, not by hollow cube corner mirror process. This situation leads to a problem that there are process errors and approximate errors in theoretical derivation of the effective angle between the reflection mirrors of the special-shaped cube corner mirror, which will be magnified by the refractive index of the glass when used to measure the roll angle. Therefore, the high-precision roll angle measurement of the glass with the opposite angle mirror cannot be achieved.

In this paper, based on the theoretical derivation of three-dimensional measurement of MCCR, we propose a method for measuring roll angle suitable for hollow cube corner reflector (HCCR). This method can be directly realized by changing the included angle relationship between the three mirrors of standard HCCR. The advantage of this method is that it is not affected by the refractive index of glass, and can achieve high precision roll angle measurement. Experiments show that when the measurement range is less than $350^{\prime \prime}$, the measurement error of roll angle is better than $15^{\prime \prime}$.

The schematic diagram of the optical path of the plane mirror is shown in Fig.1, where the solid line is the incident light, and the dashed line is the reflected light. When the plane mirror is rotated about the pitch angle $\Theta_{\mathrm{p}}$ (Fig.1(a)) and the yaw angle $\Theta_{\mathrm{y}}$ (Fig.1(b)), respectively, it can be found that the reflected beam will reflect off the angles of $2 \Theta_{\mathrm{p}}$ and $2 \Theta_{\mathrm{y}}$ around the main optical axis. Conversely, when the plane mirror rotates around the roll angle $\Theta_{\mathrm{r}}$ (Fig.1(c)), the reflected beam will not shift. It can be seen from the figure that when the plane mirror rotates around the roll axis, it will not cause the deflection of the optical axis of the reflected beam, so the roll angle cannot be measured. We have designed an HCCR, which can effectively convert the roll angle information into the spot position information on the detector and realize the measurement of roll angle.

(a)

(b)
(c)

Fig. 1 Schematic diagram of the optical path of the plane mirror: (a) $O X$ as the rotation axis; (b) $O Y$ as the rotation axis; (c) $O Z$ as the rotation axis

According to Fig. 6 of Ref.[15], the structure of MCCR used for large-range three-dimensional measurement is shown in Fig.2. The MCCR is composed of mirror 1, 2 and 3 , respectively, and the mirror 1 is cut into 1 a and 1 b . The pitch angle $\Theta_{1}$ and yaw angle $\Theta_{2}$ are measured by the beams with reflection order of 3-2-1a and 1a-2-3, and the roll angle $\Theta_{3}$ is measured by the beams with reflection order of 1b-3-2 and 2-3-1b.


Fig. 2 Structure diagram of MCCR
Among them, $\delta_{1}, \delta_{2}, \delta_{3}$ represent the difference values between the angle between planes 1 and 2, planes 2 and 3 , and between planes 1 and 3 and $90^{\circ}$, respectively, and $\delta_{12}$ is the angle between 1a and 1b. According to Eq.(12) in Ref.[15], its expression can be deduced as

$$
\begin{equation*}
\delta_{1}=2 \delta^{2}, \tag{1-1}
\end{equation*}
$$

$\delta_{2}=\delta$,
$\delta_{3}=-\delta$,
where $\delta$ represents the effective structura ang MCCR. When the non-standard MCCR satisfies this measurement relationship, the roll angle $\Theta_{3}$ can be measured, and the roll angle $\Theta_{3}$ measurement formula can be derived according to the Eqs.(24), (27) and (30) in Ref.[15] as

$$
\begin{equation*}
X_{1 \mathrm{lb} 32}=f \cdot\left(-2 \sqrt{2} \delta \cdot n \cdot \Theta_{3}\right), \tag{2}
\end{equation*}
$$

where $n$ is the refractive index of MCCR. When $\delta \leq 1^{\circ}$, according to Eqs.(24), (27) and (30) in Ref.[15], the roll angle measurement Eq.(2) can be derived. The approximate error of $\delta$ is about $0.5 \%$ due to the approximate processing formulas. According to Eq.(2), the readout value of the roll angle $\Theta_{3}$ of the MCCR is calculated by converting the measured value of spot displacement $X_{1 \mathrm{~b} 32}$. However, due to the approximate error of $\delta$, the measurement accuracy will be affected. In addition, in the
actual production process, the effective angle between the reflection mirrors of MCCR has a process error of about $\pm 5^{\prime \prime}$. And the refractive index amplifies these two errors by a factor of $n$. As a result, the root mean square ( $R M S$ ) value of the measurement error is $40.43^{\prime \prime}$ in the process of measuring the roll angle of the autocollimator with MCCR. In order to improve the measurement accuracy of roll angle, non-standard hollow mirror technology can be used to avoid the influence of refractive index on the measurement.

According to the structure of the MCCR designed in Ref.[15], it is mainly divided into two parts of the measurement structure. In order to achieve the measurement of the roll angle, we retain the performance of measuring the roll angle in Fig. 2 and design an HCCR structure for roll angle measurement. And the angular relationship of the HCCR is shown in Fig.3. According to the rotating structure shown in Fig. 3 in Ref.[15], the angle between the planes 1 and 2 is $\angle 1 \_2=90^{\circ}-\delta_{1}$, the angle between the planes 2 and 3 is $\angle 2 \_3=90^{\circ}-\delta_{2}$, and the angle between the planes 1 and 3 is $\angle 1 \_3=90^{\circ}-\delta_{3}$, while $\delta_{12}=\delta$, $\delta_{2}=\delta_{23}, \delta_{3}=\delta_{13}$.


Fig. 3 Structure diagram of HCCR
According to Eqs.(3) and (10) in Ref.[15] and the coordinate systems shown in Figs. 4 and 5 in Ref.[15], the reflection matrix can be expressed as shown in Eq.(3), and the unit vector $\boldsymbol{U}$ is shown in Eq.(4).

$$
\boldsymbol{M}_{\mathrm{d}}=\left(\begin{array}{ccc}
1-2 \cdot N_{x}^{2} & -2 \cdot N_{x} \cdot N_{y} & -2 \cdot N_{x} \cdot N_{z}  \tag{3}\\
-2 \cdot N_{x} \cdot N_{y} & 1-2 \cdot N_{y}^{2} & -2 \cdot N_{z} \cdot N_{y} \\
-2 \cdot N_{x} \cdot N_{z} & -2 \cdot N_{z} \cdot N_{y} & 1-2 \cdot N_{z}^{2}
\end{array}\right),
$$

where $N_{x}, N_{y}$, and $N_{z}$ are the coordinates of the unit vectors at the mirror projections perpendicular to the plane mirror reflector.

$$
\boldsymbol{U}=\left(\begin{array}{c}
-\frac{1}{\sqrt{2}}  \tag{4}\\
\frac{1}{\sqrt{2}} \\
0
\end{array}\right)
$$

For the measurement of roll angle $\Theta_{3}$, beams with reflection order of 1-3-2 and 2-3-1 were selected for research, and the corresponding reflection matrices were
$\boldsymbol{M}_{132}$ and $\boldsymbol{M}_{231}$, respectively.

$$
M_{132}=\left(\begin{array}{lll}
a_{3} & b_{3} & c_{3} \tag{5}
\end{array}\right),
$$

where

$$
\boldsymbol{a}_{3}=\left(\begin{array}{c}
1-2 \cos \left(\delta_{3}\right)^{2} \cdot \cos \left(\delta_{12}\right)^{2}  \tag{6}\\
\sin \left(2 \delta_{12}\right) \cdot \cos \left(\delta_{3}\right)^{2} \\
\sin \left(2 \delta_{3}\right) \cdot \cos \left(\delta_{12}\right)
\end{array}\right),
$$

$\boldsymbol{b}_{3}=\left(\begin{array}{c}-\cos \left(2 \delta_{2}\right) \cdot \sin \left(2 \delta_{12}\right) \cdot \cos \left(\delta_{3}\right)^{2}-\sin \left(2 \delta_{2}\right) \cdot \sin \left(2 \delta_{3}\right) \cdot \cos \left(\delta_{12}\right) \\ \cos \left(2 \delta_{2}\right) \cdot\left[2 \cos \left(\delta_{3}\right)^{2} \cdot \sin \left(\delta_{12}\right)^{2}-1\right]+\sin \left(2 \delta_{2}\right) \cdot \sin \left(2 \delta_{3}\right) \cdot \sin \left(\delta_{12}\right) \\ \cos \left(2 \delta_{2}\right) \cdot \sin \left(2 \delta_{3}\right) \cdot \sin \left(\delta_{12}\right)-\cos \left(2 \delta_{3}\right) \cdot \sin \left(2 \delta_{2}\right)\end{array}\right)$,
$\boldsymbol{c}_{3}=\left(\begin{array}{c}\sin \left(2 \delta_{2}\right) \cdot \sin \left(2 \delta_{12}\right) \cdot \cos \left(\delta_{3}\right)^{2}-\cos \left(2 \delta_{2}\right) \cdot \sin \left(2 \delta_{3}\right) \cdot \cos \left(\delta_{12}\right) \\ \cos \left(2 \delta_{2}\right) \cdot \sin \left(2 \delta_{3}\right) \cdot \sin \left(\delta_{12}\right)-\sin \left(2 \delta_{2}\right) \cdot\left[2 \cos \left(\delta_{3}\right)^{2} \cdot \sin \left(\delta_{12}\right)^{2}-1\right] \\ -\cos \left(2 \delta_{2}\right) \cdot \cos \left(2 \delta_{3}\right)-\sin \left(2 \delta_{2}\right) \cdot \cos \left(2 \delta_{3}\right) \cdot \sin \left(\delta_{12}\right)\end{array}\right)$.

Now, let the unit vector $\boldsymbol{C}$ enter from the opposite direction of the unit vector $\boldsymbol{U}$ parallel to the reference axis of the rotating object.
$\boldsymbol{C}=-\boldsymbol{U}$.
To ensure that the reflection vector $\boldsymbol{B}$ is parallel to vector $\boldsymbol{C}$ and in the opposite direction to $\boldsymbol{C}, \boldsymbol{B}$ should satisfy the following formula.

$$
\begin{equation*}
\boldsymbol{B}=\boldsymbol{M}_{132} \cdot \boldsymbol{C}=\boldsymbol{M}_{132} \cdot(-\boldsymbol{U})=\boldsymbol{U} . \tag{10}
\end{equation*}
$$

According to Eqs.(1), (2) and (10), the reflection matrix $\boldsymbol{M}_{132}$ can be expressed as

$$
\boldsymbol{M}_{132}=\left(\begin{array}{ccc}
-1+2 \delta^{2} & 2 \delta^{2} & 2 \delta  \tag{11}\\
2 \delta^{2} & -1+2 \delta^{2} & 2 \delta \\
-2 \delta & -2 \delta & -1+4 \delta^{2}
\end{array}\right)
$$

Since the reflection order of the reflected light beams 2-3-1 and 1-3-2 are opposite, in the reflection matrices $\boldsymbol{M}_{231}$ and $\boldsymbol{M}_{132}$, except for the main diagonal elements, other elements are in opposite numbers.

The relation between the reflection matrix in the coordinate system $X Y Z$ and that in the coordinate system $X_{1} Y_{1} Z_{1}$ can be expressed as

$$
\begin{equation*}
\boldsymbol{M}_{132}=\boldsymbol{R}_{\mathrm{t}} \cdot \boldsymbol{M}_{0-132} \cdot \boldsymbol{R}_{\mathrm{t}}^{\mathrm{T}}, \tag{12}
\end{equation*}
$$

among which, $\boldsymbol{M}_{132}$ and $\boldsymbol{M}_{0-132}$ are the reflection matrices of coordinate system $X_{1} Y_{1} Z_{1}$ and $X_{0} Y_{0} Z_{0}$ respectively, $\boldsymbol{R}_{\mathrm{t}}$ is the transformation matrix from coordinate system $X_{0} Y_{0} Z_{0}$ to coordinate system $X_{1} Y_{1} Z_{1}$, and the transformation angles corresponding to $X-, Y$ - and $Z$ - are $\varphi, \nu$, and $\psi$ respectively. When $\varphi=\arcsin (\sqrt{2} / \sqrt{3}), \quad \nu=0$, and $\psi=-3 \pi / 4, \boldsymbol{R}_{\mathrm{t}}$ can be expressed as

$$
\boldsymbol{R}_{\mathrm{t}}=\left(\begin{array}{ccc}
-\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0  \tag{13}\\
\frac{\sqrt{2} \cdot \sqrt{3}}{6} & \frac{\sqrt{2} \cdot \sqrt{3}}{6} & \frac{\sqrt{2} \cdot \sqrt{3}}{3} \\
\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3}
\end{array}\right) .
$$

Therefore, the reflection matrix $\boldsymbol{M}_{132}$ in Eq.(11) can be expressed as

$$
\boldsymbol{M}_{132}=\left(\begin{array}{ccc}
-1 & 0 & 0  \tag{14}\\
0 & 4 \delta^{2}-1 & -2 \sqrt{2} \delta \\
0 & 2 \sqrt{2} \delta & 4 \delta^{2}-1
\end{array}\right) .
$$

The unit vector $\boldsymbol{B}$ of the reflected beam is

$$
\begin{equation*}
\boldsymbol{B}=\boldsymbol{M}_{132} \cdot \boldsymbol{A}, \tag{15}
\end{equation*}
$$

where $\boldsymbol{A}$ is the incident beam entering the HCCR, and $\boldsymbol{A}=(0,0,1)^{\mathrm{T}}$.

$$
\boldsymbol{B}=\left(\begin{array}{c}
0  \tag{16}\\
2 \sqrt{2} \delta \\
1-4 \delta^{2}
\end{array}\right) .
$$

When the roll angle of HCCR is $\Theta_{3}$, the unit vector $\boldsymbol{B}_{\mathrm{r}}$ corresponding to the reflected beam can be expressed as

$$
\begin{equation*}
\boldsymbol{B}_{\mathrm{r}}=\boldsymbol{M}_{132} \cdot \boldsymbol{A} . \tag{17}
\end{equation*}
$$

According to Eqs.(16) and (19), the unit vector $\boldsymbol{B}_{132}$ of the reflected beam in the order of 1-3-2 can be calculated.


The roll angle $\Theta_{3}$ can be obtained from the first column element $-2 \sqrt{2} \delta \cdot \sin \left(\Theta_{3}\right) \cdot \cos \left(\Theta_{1}\right)$ in Eq.(20).

This means that the roll angle of HCCR can be measured by using the reflected light beam with the reflection order of 1-3-2, and when the roll angle of HCCR changes, the reflected light spot on the receiving surface of the autocollimator will shift along the $X_{0}$ axis, as shown in Fig. 4.


Fig. 4 The imaging displacement of the receiving surface of the autocollimator when the HCCR rotates along the roll angle

In this section, we will introduce the experimental verification of roll angle measurement performance.

An experimental device as shown in Fig. 5 is built. According to Eqs.(11) and (12), the effective structural angles
of HCCR are designed as follows: $\delta_{1}=\delta_{3}=1.76^{\circ}, \delta_{2}=$ $-1.76^{\circ}$. The roll angle autocollimator (roll angle AC) is based on the Nikon 6D prototype, made in Japan, with a focal length of 700 mm and an aperture of 70 mm . The measuring range of vertical and horizontal axes is $30^{\prime}$ and the accuracy is $0.5^{\prime \prime}$. The model of the three-degree-offreedom turntable is PT5, made in China, with a minimum step length of 10 ", and the three-degree-of-freedom rotation range is more than $30^{\circ}$.

Digital autocollimator 1 (Ref. AC1) is CA860, produced by Anshan Guangzhun Technology Co., Ltd., with an aperture of 60 mm , a resolution of 0.01 ", and a measurement range of $\pm 350^{\prime \prime}$ with an accuracy of $0.3^{\prime \prime}$. Reference autocollimator 2 (Ref. AC2) is based on the Nikon 6D prototype, made in Japan, with a focal length of 700 mm and an aperture of 70 mm . The measuring range of vertical and horizontal axes is $30^{\prime}$ and the accuracy is 0.5 ".

In order to objectively evaluate the measurement performance of the roll angle autocollimator, Ref. AC1 and Ref. AC 2 are used as the measurement datum. According to the position (as shown in Fig.5), the Ref. AC1 can measure the roll angle of the turntable, while the Ref. AC 2 can monitor the rotation state of the turntable, that is, to ensure that the turntable rotates only along the roll angle. Ref. AC1 and Ref. AC2 are aligned with the two coated orthogonal surfaces of the glass cube respectively. Such a measurement reference can ensure that the Ref. AC 1 and Ref. AC2 are orthogonal to each other, so as to avoid the cross-talk between the $X_{0}$ and $Y_{0}$ axes of the receiving surfaces of the Ref. AC1 and Ref. AC2.


Fig. 5 Photograph of the experimental setup
The traditional formula of autocollimator measurement is as follows

$$
\begin{equation*}
X=K \cdot f \cdot \tan (\Theta), \tag{19}
\end{equation*}
$$

where $X$ is the displacement of the reflected light spot, $f$ is the focal length of the autocollimator, $K$ is the reflection
angle sensitivity of the reflector (for a plane mirror, $K=2$ ), and $\Theta$ is the yaw and pitch angle. In order to obtain the measurement formula of the roll angle autocollimator, it is necessary to determine the reflection angle sensitivity $K_{\text {HCCR }}$ of HCCR. Its measurement method is as follows.
(1) The turntable rotates around the $Z_{1}$ axis, the rotation angle is the actual display angle of the Ref. AC1, with a step length of $25^{\prime \prime}$, and the range is $0-350^{\prime \prime}$, which is recorded as the actual rotation value $\Theta_{3 K}$ of the HCCR
(2) The read value of step (1) is the read value when the Ref. AC 2 is always at the zero position.
(3) Read out the reading value $\Theta_{3}$ of the angle measured by the roll angle autocollimator under each step size change.
(4) Calculate the ratio of the read value $\Theta_{3}$ of the roll angle autocollimator to the actual rotation value $\Theta_{3 K}$ of the HCCR, and obtain the reflection angle sensitivity of the HCCR according to Eq.(20).
(5) To verify the repeatability of the reflection angle sensitivity test, the above process was repeated 6 times.

$$
\begin{equation*}
K_{\mathrm{MCCR}}=K \cdot \frac{\Theta_{3 K}}{\Theta_{3}} \tag{20}
\end{equation*}
$$

According to the actual rotation angle of HCCR and the readout data of the roll angle autocollimator, in addition, root-mean-square processing was performed on the results of the six experiments to obtain the measurement results as shown in Fig.6. It can be seen from the figure that linear fitting is performed on the measured values of the results of the 6 experiments, and the slope obtained is $\Theta_{3} / \Theta_{3 K}=0.04$. Substituting the linear slope value into Eq.(20), the reflection sensitivity $K_{\text {HCCR }}$ of HCCR calculated is 0.080 . The calculated value approximates the fixed value of $-2 \sqrt{2} \delta$ for the first order term in Eq.(18), which means that the reflectance sensitivity of HCCR will be determined by $\delta$. However, the effective structural angle of HCCR we used is $1.76^{\circ}$, so the theoretical $K_{\text {HCCR }}$ of HCCR reflectance sensitivity is 0.087 , which has a difference of 0.007 from the actual $K_{\mathrm{HCCR}}$.


Fig. 6 HCCR roll angle readout based on Nikon6D autocollimator

We will reevaluate the measurement accuracy of the roll angle autocollimator using the HCCR actual reflec-
tance sensitivity $K_{\mathrm{HCCR}}$ obtained above and the measurement Eq.(20). The specific experimental steps are similar to those above, but the difference is that in order to obtain more objective evaluation results, we reduce the sampling step size to $10^{\prime \prime}$, so as to obtain more measurement data of measurement points. By calculating the $R M S$ of experimental data for six times at each step length, the experimental results are drawn as shown in Fig.7. The experimental results show that when the measured value is $90^{\prime \prime}$, the maximum $R M S$ error is $15^{\prime \prime}$. When the measured value is 190 ", the maximum repeatability error is better than $2.7^{\prime \prime}$. The $R M S$ of the HCCR-based roll angle autocollimator is less than $15^{\prime \prime}$. The measurement error $R M S$ is 30 times of the original measurement accuracy of Nikon6D $0.5^{\prime \prime}$. This is because the reflection angle sensitivity $K_{\mathrm{HCCR}}$ of HCCR is 0.080 , while the reflection angle sensitivity of the plane mirror of the traditional autocollimator is $K=2$. However, the resolution of the autocollimator is reduced due to the reduced sensitivity of the HCCR reflection angle.


Fig. 7 Measurement results of the roll angle AC after calibration with the actual HCCR reflection angle sensitivity

This paper proposes a new design method for the measurement of the roll angle autocollimator. This is by using the Euler rotation relationship to establish the space coordinate vector relationship between the mirror and the autocollimator, and obtain an HCCR structure. Experimental results show that the HCCR can be used to measure the roll angle by replacing the plane mirror of the traditional autocollimator. Meanwhile, the reflectance sensitivity $K_{\mathrm{HCCR}}$ of HCCR structure is obtained experimentally, and the mathematical relationship between the reflectance sensitivity $K_{\mathrm{HCCR}}$ and the effective angle of HCCR is discussed by theoretical regression analysis. Based on the above work, the measurement formula based on the roll angle autocollimator is established, and the measurement performance of the proposed roll angle autocollimator is demonstrated by increasing the frequency of sampling points in the experiment. Experiments show that in the 350 " measurement range, the repeatability of the experiment is better than $2.7^{\prime \prime}$, the maximum $R M S$ error is 15 ", which indicates that the error of the autocollimator using the HCCR mirror is
expanded by 30 times. This shows that the autocollimator uses the HCCR to enlarge the error by 30 times. This is because the HCCR reduces the sensitivity of the reflection angle compared with a plane mirror, which leads to a decrease in the resolution of the autocollimator. The use of a higher resolution photoelectric sensor will increase the focal length of the autocollimator, which can effectively suppress this effect.

## Statements and Declarations

The authors declare that there are no conflicts of interest related to this article.

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