Entanglement of three-level atomic system and spontaneous emission fields in a vacuum and near the 1D photonic crystal band gap

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The time evolution of the quantum entropy in a GaAs/AlAs one-dimensional photonic crystal (1DPC) with an atomic system defect layer is investigated in this work. The entanglement between atomic system and their spontaneous emission fields near the edge of the photonic band gap (PBG) is coherently controlled by the coupling field. Comparison between the atom-photon entanglement of the atomic system in the vacuum surrounding and that near the PBG of the 1DPC shows that the degree of entanglement strongly depends on the PBG. We find that degree of entanglement is strongly dependent on the intensity and detuning of the coupling and probe fields. Furthermore, the effect of the phase difference between applied fields on the atom-photon entanglement is studied. The potentially possible technological applications can be provided by the proposed model in the quantum optics and quantum communications based on photonic crystal.

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There are many phenomena in the field of quantum optics based on quantum coherence and quantum interference[1], but quantum entanglement has recently attracted the attention of many researchers from a practical point of view. Entanglement plays a very important role in quantum information theory, such as quantum communication, quantum computation, quantum cryptography, quantum teleportation, quantum dens codding, and so on[2]. In the two last decades, in order to produce entangled particles, various approaches were proposed, such as atom-atom, atom-photon, and photon-photon entanglement based on quantum interference[3]. As atom stores the quantum information and photon carries quantum information, atom-photon entanglement has reached special attention recently[4]. Atom-photon entanglement based on electromagnetic induced transparency (EIT) in atomic Bose-Einstein condensate[5], atom-photon entanglement of a two-level atom near the edge of a photonic band gap[6], atom-photon entanglement for a four-level atom near the band edge of a 3D-anisotropic photonic crystal (PC)[7] and atom-photon entanglement for a four-level atomic system via incoherent pumping field[8] have been investigated. It is well known that in atom-photon entanglement, the atom entangles with the photons from spontaneous emission. One of the important factors that reduce the coherence of the atomic system is spontaneous emission, which can be controlled in different ways such as selecting microwave cavity[9] as a surrounding of atoms or use of the dispersive band-gap materials such as PCs[10]. Therefore, two important factors influencing spontaneous emission are the energy structure of the atom and the density of the ambient photons where the atom is located[11]. The PC is one of the environments that can affect the spontaneous emission of the atom due to the band gap[12], and the different density of the states of the background photons from the vacuum and other environments[13]. On the other hand, the PC is an environment with an alternating structure of dielectrics that can control the transmission of light according to its structure. Therefore, due to the controllable parameters of the PC, such as dielectric materials, layer structure and defect layer, and having a band gap, it allows us to control propagation of electromagnetic waves and densities of states of the radiation field that are in the band gap range[14].

In this paper, we investigate the atom-photon entanglement of the three-level atomic system, which is considered as a defect layer in a one-dimensional photonic crystal (1DPC). According to the application perspectives, 1DPC based on GaAs/AlAs is considered as the environment around atoms. We compared the results with atom-photon entanglement in vacuum surrounding. We study the effect of the detuning, intensity, and phase difference of the coupling and probe fields on the dynamic behavior of the

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atom-photon entanglement near the photonic band gap (PBG). To investigate the degree of atom-photon entanglement, we consider the temporal evolution of the quantum entropy of the quantum system.

The considered model is a 1DPC based on GaAs/AlAs with a defect layer that includes a three-level Λ-type atomic system as shown in Fig.1. The optical thicknesses of dielectric structure of the 1DPC are d_d=800 nm, d_{GaAs}=286 nm and d_{AlAs}=362 nm. According to Ref.[15], we select the refractive indexes of the GaAs and AlAs as n_{GaAs}=3.4 and n_{AlAs}=2.9.

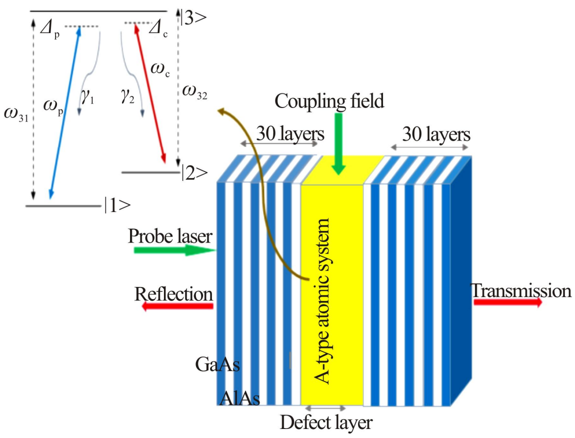


Fig.1 Cross-section of 1DPC based on GaAs/AlAs with a three-level Λ-type defect layer

Reflective index of the defect layer is given by the linear electric susceptibility ($\chi(\omega)$) of the atomic system $n_D(\omega)=\sqrt{\varepsilon_B+\chi(\omega)}$, where ε_B=1 is the background dielectric function and $\chi(\omega)$ is calculated based on density matrix operator. According to the schematic of the D2 transition ($5^2P3/2$-$5^2S1/2$) of the ^{87}Rb atomic system as shown in Fig.2, probe and coupling fields are applied on the three-level Λ-type quantum systems in the defect layer between levels of $|1\rangle\leftrightarrow|3\rangle$ and $|2\rangle\leftrightarrow|3\rangle$,

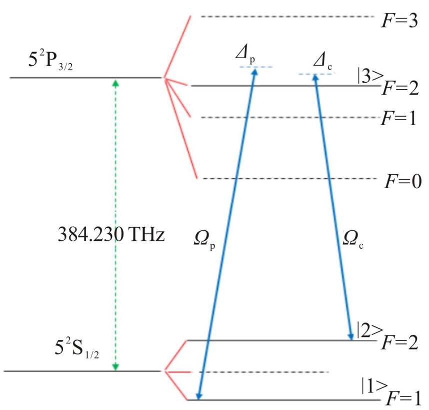


Fig.2 Schematic of D2 transition hyperfine energy structure of the ^{87}Rb, with probe filed (Ω_p) and coupling field (Ω_c) applied between levels $|1\rangle\leftrightarrow|3\rangle$ and $|2\rangle\leftrightarrow|3\rangle$, respectively

respectively. Rabi-frequencies of the probe and coupling fields are given by $\Omega_p=\frac{\vec{E}_p.\vec{\mu}_{13}}{2\hbar}$ and $\Omega_c=\frac{\vec{E}_c.\vec{\mu}_{23}}{2\hbar}$, respectively, where, $\vec{E}$ is the amplitude of light and $\vec{\mu}$ is the electric dipole moment.

The total Hamiltonian expressions of the interaction of applied fields and background's radiation fields with atomic system are given as

$$H_0=-\hbar[\omega_1|1\rangle\langle 1|+\omega_2|2\rangle\langle 2|+\omega_3|3\rangle\langle 3|], \tag{1}$$

$$H_1=-\hbar[\Omega_p e^{-i\nu_p t}|1\rangle\langle 3|+\Omega_c e^{-i\nu_c t}|2\rangle\langle 3|+H.C], \tag{2}$$

$$H=H_0+H_1, \tag{3}$$

$$V=-\hbar\sum_k g_k^{(1)}e^{-i\nu_k t}|3\rangle\langle 1|a_k-\hbar\sum_k g_k^{(2)}e^{-i\nu_k t}|3\rangle\langle 2|a_k+H.C, \tag{4}$$

where g_k^i and ν_k are the coupling constant and frequency of background vacuum photons, respectively. Also, a_k and $a_k^\dagger$ are the creation and annihilation operator, respectively. The density matrix operator in order to calculate the equation of motion is given by

$$\dot{\rho}=\frac{-i}{\hbar}[H,\rho]+\dot{\rho}_{sp}, \tag{5}$$

where $\dot{\rho}_{sp}$ is the terms of the spontaneous emission and given by

$$\dot{\rho}_{sp_{kj}}=(\frac{i}{\hbar})^2 Tr\int[V(t),[V(t'),\rho(t')]]_{kj}\,dt'. \tag{6}$$

By embedment of Eqs.(3) and (4) into Eqs.(5) and (6), respectively, motion equations of the density matrix are obtained as

$$\dot{\rho}_{11}=i|\Omega_p|(\rho_{31}-\rho_{13})+2\gamma_{31}\rho_{33}, \tag{7-1}$$

$$\dot{\rho}_{22}=i|\Omega_c|(\rho_{32}-\rho_{23})+2\gamma_{32}\rho_{33}, \tag{7-2}$$

$$\dot{\rho}_{33}=i|\Omega_p|(\rho_{13}-\rho_{31})+i|\Omega_c|(\rho_{23}-\rho_{32})-2(\gamma_{31}+\gamma_{32})\rho_{33}, \tag{7-3}$$

$$\dot{\rho}_{12}=i[\Delta_c-\Delta_p]\rho_{12}+i|\Omega_p|\rho_{32}-i|\Omega_c|\rho_{13}+2e^{\Delta\varphi}\sqrt{\gamma_{31}\gamma_{32}}\rho_{33}, \tag{7-4}$$

$$\dot{\rho}_{21}=-i[\Delta_c-\Delta_p]\rho_{21}-i|\Omega_p|\rho_{23}+i|\Omega_c|\rho_{31}+2e^{-\Delta\varphi}\sqrt{\gamma_{31}\gamma_{32}}\rho_{33}, \tag{7-5}$$

$$\dot{\rho}_{13}=-i\Delta_p\rho_{13}+i|\Omega_p|(\rho_{33}-\rho_{11})-i|\Omega_c|\rho_{12}-(\gamma_{31}+\gamma_{32})\rho_{13}, \tag{7-6}$$

$$\dot{\rho}_{31}=i\Delta_p\rho_{31}-i|\Omega_p|(\rho_{33}-\rho_{11})+i|\Omega_c|\rho_{21}-(\gamma_{31}+\gamma_{32})\rho_{31}, \tag{7-7}$$

$$\dot{\rho}_{23}=-i\Delta_c\rho_{23}-i|\Omega_p|\rho_{21}+i|\Omega_c|(\rho_{33}-\rho_{22})-(\gamma_{31}+\gamma_{32})\rho_{23}, \tag{7-8}$$

$$\dot{\rho}_{32}=i\Delta_c\rho_{32}+i|\Omega_p|\rho_{12}-i|\Omega_c|(\rho_{33}-\rho_{22})-(\gamma_{31}+\gamma_{32})\rho_{32}, \tag{7-9}$$

where $\rho_{ij}=\rho_{ij}^*$ and $\rho_{11}+\rho_{22}+\rho_{33}=1$. Here, $\Delta_p=\omega_p-\omega_{13}$ is the probe light detuning and $\Delta_c=\omega_c-\omega_{23}$ is the coherence coupling field detuning. $\Delta\varphi=\varphi_p-\varphi_c$ is the phase difference between probe and coupling fields which are applied on the atomic system. In addition, spontaneous decay rates between levels $|3\rangle\rightarrow|1\rangle$ (γ_{31}) and $|3\rangle\rightarrow|2\rangle$ (γ_{32}) ($|3\rangle\rightarrow|2\rangle$) near the PBG are given as[16]

$$\gamma_1(\varepsilon_{31})=\gamma_{31}Z(\varepsilon_{31})^2, \tag{8-1}$$

$$\gamma_2(\varepsilon_{32})=\gamma_{32}Z(\varepsilon_{32})^2, \tag{8-2}$$

where ε_{ij} is the energy of the applied field between levels $|i\rangle\leftrightarrow|j\rangle$ and $Z(\varepsilon_{ij})$ is given as

$$Z(\varepsilon_{ij})=\frac{(\varepsilon_c-\varepsilon_{ij})^2}{(\varepsilon_v-\varepsilon_{ij})^2+\kappa^2}, \tag{9}$$

where ε_c and ε_v are the minimum and maximum values of the upper and lower bands of the PBG, respectively. Here κ is a constant related to relaxation processes of the surrounding of the atomic system.

There are mathematical methods for quantifying entanglement, the most important and widely used of which in quantum systems is the Van Neumann quantum entropy based on density matrix operator. A two-component system in Hilbert space $(C'''\otimes C'')$ is described by the density matrix operator. The partial density matrix of one part is given by tracing on another part:

$$\rho_{A(B)}=Tr_{B(A)}(\rho_{AB}). \tag{10}$$

A two-part quantum system is separable if it can be written as follows

$$\rho_{A(B)}=\rho_A\otimes\rho_B, \tag{11}$$

where $\rho_{A(B)}$ is the individual partial of the density matrixes. So, if the quantum system cannot satisfy Eq.(11), it is said to be entangled. We use the evolution of Von Neumann entropy to estimate the degree of entanglement between atoms and their spontaneous emission photons. ARAKI et al[17] presented a substantial theorem to measurement entropy of tow-component quantum system given as

$$|S_A(t)-S_F(t)|\le S_{AF}\le S_A(t)+S_A(t), \tag{12}$$

where $S_{AF}=-Tr(\rho^I_{AF}\ln\rho^I_{AF})$ indicates the time evolution of total entropy of the composite system of atoms and their emission photons, where $\rho^I_{AF}(t)$ is given by Eq.(10). We assume initially and before interaction between atoms and spontaneous fields, the three-level quantum system in the defect layer starts from the disentangled pure state. So, based on this assumption, the total entropy of the system of rubidium atoms-spontaneous decay fields is zero, which means $S_A(t)=S_F(t)$. We need to calculate the entropy of the three-level atoms $S_A(t)$ to discuss the time evolution of entanglement degree. The reduced entropy of the rubidium atomic vapor as a defect layer of 1DPC can be expressed by reduced-density operators[18]

$$S_A(t)=-Tr(\rho_A\ln\rho_A), \tag{13}$$

where ρ_A is the reduced density operator of the three-level atomic vapor that is calculated by elements of Eq.(11). Therefore, the degree entanglement of the three-level atom-photon quantum system based on entropy can be defined by

$$S_A(t)=-\sum_{A=1}^{3}\lambda_d(t)\ln(t)\lambda_d(t), \tag{14}$$

where $\lambda_d(t)$ is the eigenvalues of the reduced density matrix of the three-level rubidium atomic system.

The results for the dynamic behavior of the quantum entropy of Λ-shape atomic system can be described as follows. We discuss our result for the degree of atom-photon entanglement of three level atomic system in a vacuum and near the 1DPC. In this work, all of the parameters are chosen based on the decay rates through scaling $\gamma_0=2\pi\times6.065$ MHz and all of the figures are plotted in the unit of γ_0. Based on our proposed 1DPC structure, the values of the ε_c and ε_v are 1.601 eV and 1.567 eV, respectively, and κ=0.016 meV[19].

The transmission and reflection coefficients (T and R) of 1DPC versus probe wavelength are shown in Fig.3. In this figure, the transmission and reflection of the probe light and wavelength range of the PBG are shown for the proposed 1DPC structure.

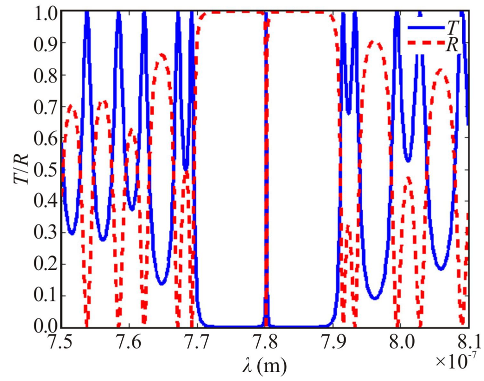


Fig.3 Transmission and reflection coefficients of 1DPC with d_{GaAs}=286 nm, d_{AlAs}=362 nm, d_{Defect}=800 nm, n_{GaAs}=3.4, n_{AlAs}=2.9, λ_0=780 nm

Fig.4 shows the dynamic behavior of the entropy for three different values of intensity of probe field, and other parameters are $\Delta_c=\Delta_p=0$ and $\Omega_c=0.25\gamma_0$. Fig.4(a) and Fig.4(b) are the time evolutions of the atom-photon entanglement in the vacuum and in the 1DPC, respectively. We can see that the quantum entropy is very sensitive to the value of the probe intensity. As the intensity of the probe light increases, the entropy of the system increases. By comparing Fig.4(a) and Fig.4(b), we can find that the degree of atom-photon entanglement in the 1DPC is higher than that in a vacuum.

In Fig.5, by selecting the parameters as $\Omega_c=0.25\gamma_0$, $\Delta_p=0$ and $\Delta_c=0.2\gamma_0$, we can clearly observe that by increasing the probe field intensity, quantum entropy of the system increases. But the result of the value of the entropy for this case, for each of the probe light intensity, is less than the results of Fig.4. Furthermore, in this case, we can show that degree of the entanglement in 1DPC (Fig.5(b)) is higher than that in vacuum (Fig.5(a)).

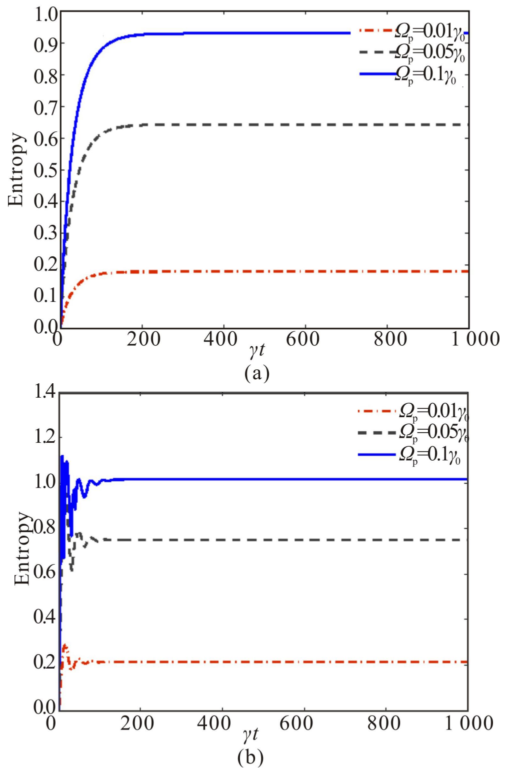


Fig.4 Dynamical behavior of the quantum entropy of the ^{87}Rb atoms (a) in the vacuum and (b) near the PBG for various values of the Rabi-frequency of probe field (Ω_c=0.25γ_0, Δ_p=Δ_c=0)

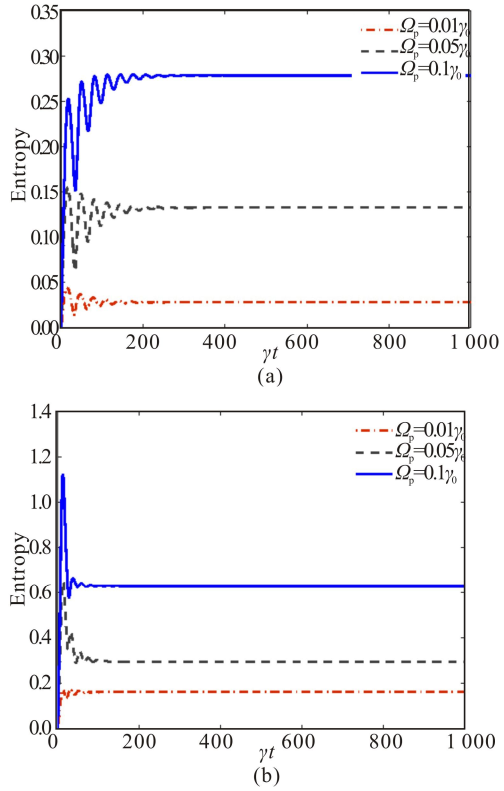


Fig.5 Dynamical behavior of the quantum entropy of the ^{87}Rb atoms (a) in the vacuum and (b) near the PBG for various values of the Rabi-frequency of probe field (Ω_c=0.25γ_0, Δ_p=0, Δ_c=0.2γ_0)

Fig.6(a) shows the effect of the phase difference between applied fields on the quantum entropy in the vacuum via selected parameters as Ω_c=0.01γ_0, Ω_p=0.01γ_0, Δ_p=0 and Δ_c=0. In this case, we see that the entropy of the system strongly depends on the phase difference, so that with increasing phase difference, the entropy increases as shown in Fig.6(a). Then, we used the same parameters to obtain the result in the 1DPC as shown in Fig.6(b). In this case, the entropy is highly sensitive to phase difference, and also its values of entropy are higher than the entropy of the atomic vapor in a vacuum (Fig.6(a)).

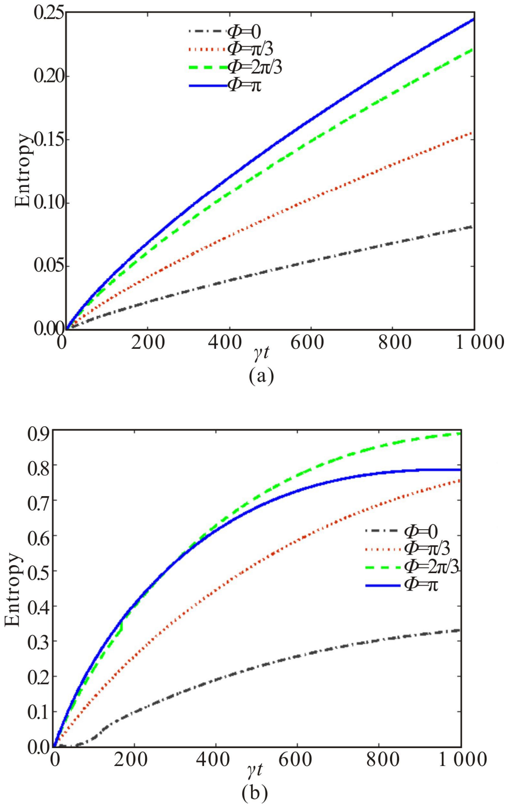


Fig.6 Time evolution of the quantum entropy of the ^{87}Rb atoms (a) in the vacuum and (b) near the PBG for various phase differences between applied fields (Ω_c=Ω_p=0.01γ_0, Δ_p=Δ_c=0)

By changing the value of the Rabi-frequency from Ω_c=0.01γ_0 to Ω_c=0.08γ_0, the entropy of the system changes slightly with increasing the phase difference as shown in Fig.7. Fig.7(b) shows that the dynamical behavior of the quantum entropy in the 1DPC stabilizes earlier than that in a vacuum (Fig.7(a)).

Fig.8 shows the dynamic behavior of the entropy for two different coupling field intensities. In this case, we can see that the quantum entropy is very sensitive to the value of the Rabi-frequency of the coupling field. The parameters which are selected for this case are Ω_p=0.001γ_0 and Δ_c=Δ_p=0. By increasing the coupling field intensity, quantum entropy of the system decreases in both the vacuum (Fig.8(a)) and 1DPC (Fig.8(b)). But in the 1DPC as a surrounding medium for the atomic vapor, the time evolution of the entropy is stabilizing earlier than vacuum.

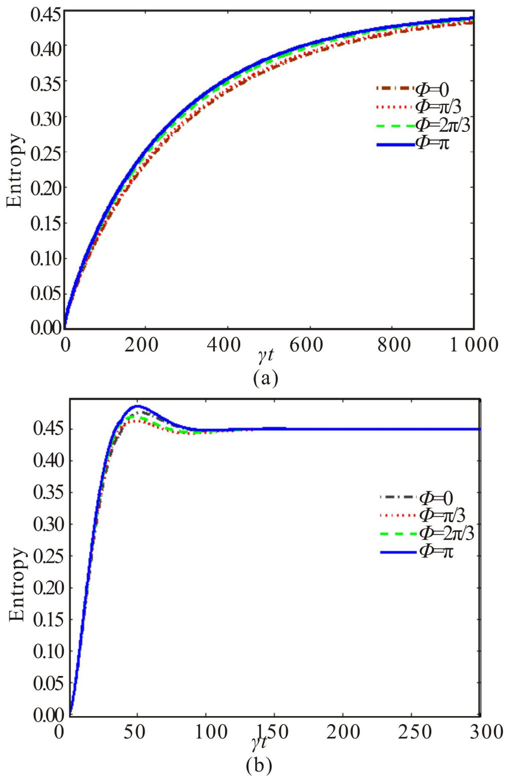


Fig.7 Time evolution of the quantum entropy of the [87]Rb atoms (a) in the vacuum and (b) near the PBG for various phase differences between applied fields (Ω_p=0.01γ_0, Ω_c=0.08γ_0, Δ_p=Δ_c=0)

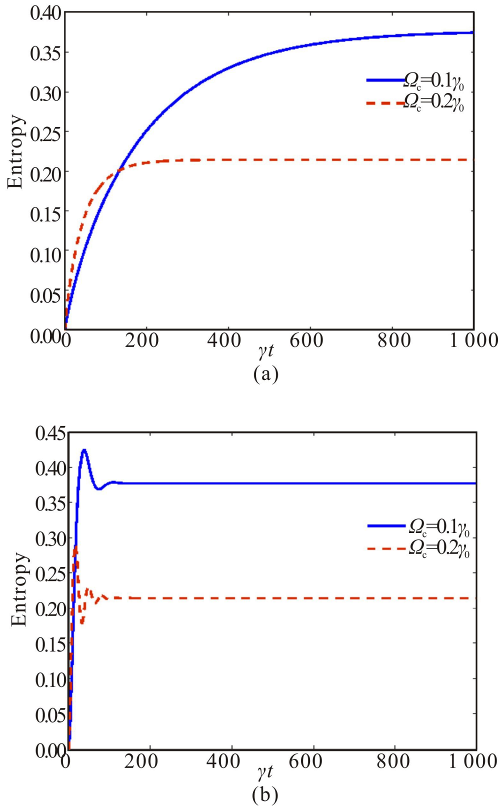


Fig.8 Dynamical behavior of the quantum entropy of the [87]Rb atoms (a) in the vacuum and (b) near the PBG for various coupling field intensities (Ω_p=0.01γ_0, Δ_p=Δ_c=0)

Fig.9 shows the effect of the Rabi-frequency via coupling field detuning. In fact, we added the parameter Δ_c=0.2γ_0 in the previous case and fixed the other parameters at the values of Δ_p=0, Ω_p=0.01γ_0 and Ω_c=(0.1γ_0, 0.2γ_0). So, Fig.9 shows that the quantum entropy of the system increases as the coupling field intensity increases. Furthermore, according to the result of Fig.9(b), degree of the entanglement in the 1DPC efficiently more increases than of that in the vacuum medium.

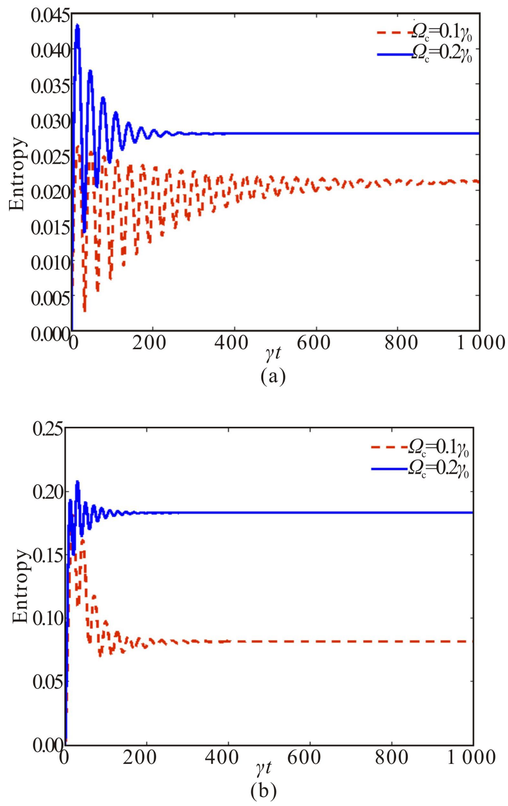
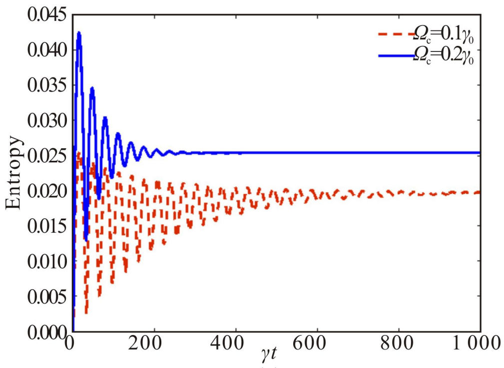
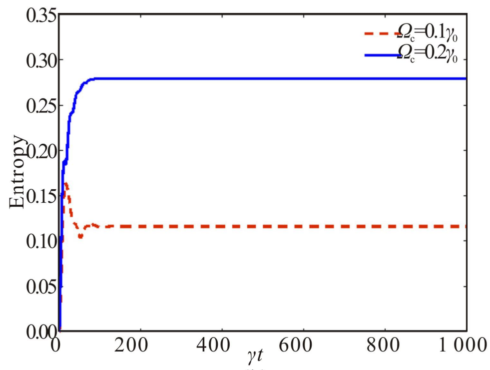


Fig.9 Dynamical behavior of the quantum entropy of the [87]Rb atoms (a) in the vacuum and (b) near the PBG for various coupling field intensities (Ω_p=0.01γ_0, Δ_p=0, Δ_c=0.2γ_0)

In Fig.10, we show the effect of the probe field detuning via two different values of the coupling field intensity. In this case, we can easily see that by applying the parameter of Δ_p=0.2γ_0 and fixing the other parameters at the values of Δ_c=0 and Ω_p=0.01γ_0, the quantum entropy of the system increases as the coupling field intensity increases. Comparing Fig.10(a) and Fig.10(b), we find that the value of quantum entropy in the 1DPC by applying the parameter of Δ_p=0.2γ_0 is greater than that in the vacuum medium. Furthermore, by comparing the results of Figs.8, 9 and 10, we find that adding parameter Δ_p or Δ_c caused the quantum entropy decreasing.



(a)

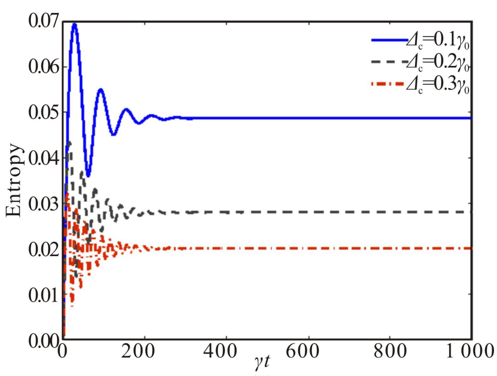


(b)

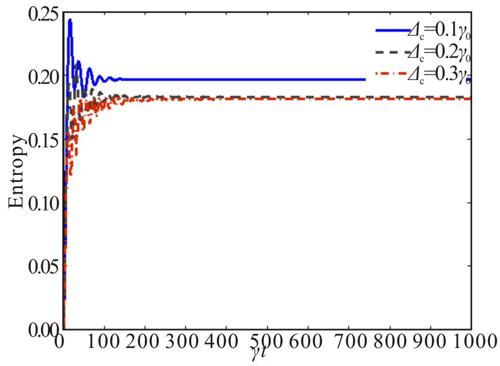
Fig.10 Dynamical behavior of the quantum entropy of the ^{87}Rb atoms (a) in the vacuum and (b) near the PBG for various coupling field intensities (Ω_p=0.01γ_0, Δ_p=0.2γ_0, Δ_c=0)

In Fig.11, the time evolution of the quantum entropy is plotted for various coupling field detuning. Fig.11(a) and Fig.11(b) depict the effect of the Δ_c on the entropy in the vacuum and 1DPC, respectively. In this case, we can show that by increasing the coupling field detuning, the time evolution of the quantum entropy of the system decreases. Therefore, by increasing the Δ_c degree of the entanglement between rubidium atoms, their spontaneous emission decays increase as shown in Fig.11.

The dynamical behavior of the atom-photon entanglement between atomic system and their spontaneous emission fields in the GaAs/AlAs 1DPC is explored. We described the degree of entanglement based on quantum entropy of the system in the vacuum and in the 1DPC through some controllable parameters. The effect of



(a)



(b)

Fig.11 Dynamical behavior of the quantum entropy of the ^{87}Rb atoms (a) in the vacuum and (b) near the PBG for various values of the coupling field detuning (Ω_p=0.01γ_0, Ω_c=0.2γ_0, Δ_p=0)

the controlling parameters, such as intensity, detuning, and phase difference between applied fields, are investigated. We find that degree of atom-photon entanglement is sensitive to phase difference and value of the Rabi-frequency and fields detuning. Also, we show that degree of the entanglement in the 1DPC is stabilized and higher than that in the vacuum.

Statements and Declarations

The authors declare that there are no conflicts of interest related to this article.

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