A calibration method of external parameters of 2D laser in rotary 3D scanning*

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Aiming at the problem of external parameter calibration in the combined rotating three-dimensional (3D) scanning process of laser radar and turntable, a fast 3D data scanning and accurate equipment calibration method suitable for industrial application scenarios is proposed. The coordinates of the center of the circle in the laser radar coordinate system are obtained by fitting the coordinates of the center of the circle with the scanning arc. Through the standard size and rotation angle of the target ball, the coordinates of the ball center in the turntable coordinate system are obtained. The rotation parameter $\boldsymbol{R}$ and the translation parameter $\boldsymbol{T}$ of the minimum value of the data set conversion error function in the two coordinate systems are calculated. The experimental results show that the error between the converted data and the actual data is within 0.1 mm, which meets the actual work requirements. This calibration method has the advantages of high accuracy and strong robustness.

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Laser radar scanner is a combination of laser technology and radar technology. It has the advantages of high resolution, good concealment and stronger anti-jamming ability. With the continuous development of science and technology, laser radar scanners are widely used in robotics, unmanned driving, mapping, cultural relics protection and other fields. At present, laser radar scanners used for ranging are mainly divided into two forms, namely two-dimensional (2D) and three-dimensional (3D) ones. The 3D laser scanner is composed of multi-line laser and rotating machinery to collect the appearance, coordinates and other information of the object. The 2D laser scanner scans the single line laser scanning surface to obtain the 2D information. Because the high price of the 3D laser scanner is a big obstacle, the 2D laser scanner is often used in the laboratory to add a rotating disc to obtain the 3D information. Before obtaining the 3D point cloud information, it is necessary to calibrate the external parameters of the laser radar and the turntable. The coordinate information of the object in the laser radar coordinate system is converted to the turntable coordinate system. In the experiment, the relative positions of the laser radar and the turntable remain unchanged. It is difficult to achieve the conversion accuracy through manual measurement. The installation position error of the turntable will also affect the external parameter solution. Therefore, it is necessary to calibrate the specific calibration object in combination with the parameter solution algorithm.

In recent years, many scholars have studied the calibration of external parameters of laser radar. HUANG et al[1] proposed a linear calibration method of system parameters through specific structural calibration parts, which uses the linear relationship between the scanning system and calibration parts to establish a linear equation group to solve the external parameters. CHENG et al[2] used the random sampling consistency algorithm to perform the optimal fitting on the set three planes, extracted the homonymous vectors and points using the optimal results, and solved the rotation and translation parameters based on the 3D coordinate system transformation model of space vectors. WU et al[3] proposed the concept of centripetal vector to eliminate the poor initial value points by obtaining two sets of data sets of the standard target ball under the online structured light vision sensor and the robot base coordinate system. Finally, the two sets of data were registered iteratively to solve the rotation translation matrix. CHENVIP et al[4] proposed a distributed automatic

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calibration algorithm. In the first step, the levelness function is constructed by fitting the ground equation, and the particle swarm optimization algorithm is used to optimize the levelness function to solve some external parameters. In the second step, the cluster center of the calibration rod is solved by the clustering algorithm and the rotation angle parameters are solved by fitting the operation line equation. After obtaining the 3D point cloud information of the workpiece by the rotary method combining the laser radar and the rotary table in the laboratory, it is necessary to process the point cloud information and transfer the coordinate information of the workpiece feature points based on the rotary table coordinate system to the next level actuator, such as the grinding robot[5]. Since the coordinate information obtained by the laser radar is in the laser radar coordinate system, before obtaining the 3D information of the workpiece, it is necessary to solve and convert the position relationship between the laser radar and the turntable to obtain the 3D point cloud information of the workpiece in the turntable coordinate system.

The calibration objects in this paper are 2D laser and rotary table. Laser can accurately measure the size and distance of objects through small laser spots. The 3D point cloud information is obtained by rotating the turntable. In order to calibrate the laser and the turntable, the laser center O_0 is selected as the origin of the laser coordinate system, the left moving direction along the cross is Z_0 axis direction, the vertical upward direction is Y_0 axis direction, and the forward moving direction along the cross is X_0 direction. Select the circle center O_1 of the turntable as the origin of the turntable coordinate system, the vertical upward direction is Z_1 axis direction, the right moving direction along the cross is Y_1 axis direction, and the forward moving direction along the cross is X_1 direction[6]. The coordinate system setting is shown in Fig.1.

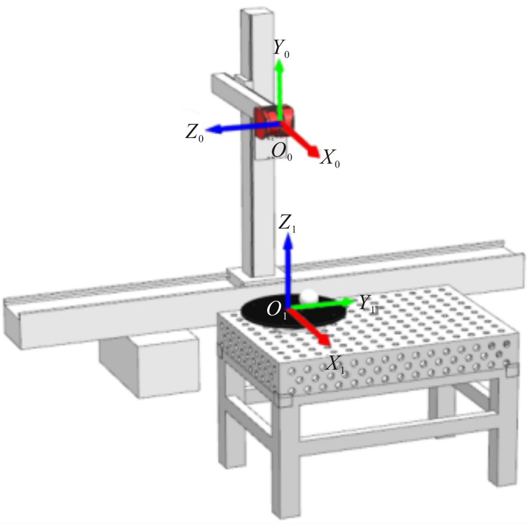


Fig.1 Coordinate system setting of the scanning system

External parameter calibration is to convert the laser radar coordinate system to the turntable coordinate system, which is represented by (X_0, Y_0, Z_0) and (X_1, Y_1, Z_1). The coordinate system transformation process is to solve the rotation parameter $\boldsymbol{R}$ and the translation parameter $\boldsymbol{T}$[7,8], namely

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \boldsymbol{R} \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + \boldsymbol{T}. \tag{1}$$

The $\boldsymbol{R}$ and $\boldsymbol{T}$ parameters are determined by solving the conversion relationship between the data set $\boldsymbol{N}=\{\boldsymbol{N}_i|\boldsymbol{N}_i\in R_3, i=1, 2, 3,..., n\}$ collected in the laser radar coordinate system and the data set $\boldsymbol{M}=\{\boldsymbol{M}_i|\boldsymbol{M}_i\in R_3, i=1, 2, 3,..., n\}$ collected in the turntable coordinate system. The key problem lies in the collection of two sets of data sets and the solution of the transformation relationship[9]. When the laser radar collects data, it emits a fan-shaped plane. When the laser collides with the measured object, it returns the distance and relative angle information between the measured object and the scanner.

In this paper, the target ball center O_A is selected as the target point, and the starting position of the target ball rotation is a point on the straight line in the positive direction of the Y_1 axis of the rotary table coordinate system whose distance to the origin O_1 is known. The rotary table drives the target ball to rotate counterclockwise to obtain the data set of different position information of the ball center in the laser radar coordinate system. However, the laser radar can only reflect the surface information of the object and cannot directly measure the spherical center. Therefore, it needs to convert from the obtained known information. When the rotating target ball passes through the scanning plane, it can scan an arc. The arc is fitted into a circle by using the fitting circle algorithm. The coordinates of the fitting circle center O_B are (X_B, Y_B, Z_B), and the coordinates of the spherical center are expressed by (X_A, Y_A, Z_A). Because the accuracy of the fitted center coordinates will affect the solution of the conversion parameters, different circle fitting methods are compared and analyzed to solve the accurate center coordinates.

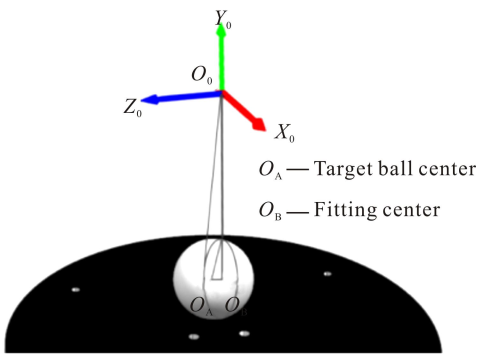


Fig.2 Intersection arc between laser radar scanning plane and target sphere

The commonly used circle fitting methods mainly include least square method and random sampling consistent algorithm[10]. Comparing the fitting principle of the algorithm, it is found that the random sampling consistent algorithm can fit the circle through the inner points, but the correctness of the fitting circle depends on the screened inner points, while the least square method will comprehensively consider all data points for fitting. In this paper, the precision and accuracy of circle fitting are highly required, so the least square method is selected. The least square fitting method is divided into algebraic fitting and geometric fitting. Comparatively speaking, the algebraic fitting method has a closed solution, is easy to solve, and has high computational efficiency, while the geometric fitting method has the advantages of better accuracy, robustness and coordinate transformation invariance. Therefore, the geometric fitting method is selected according to the requirements of accuracy. Circle fitting is a nonlinear problem. In this paper, the Gauss Newton iterative algorithm is used to solve the regression coefficient of the nonlinear regression model, and then the least square method is used for calculation. The optimal regression coefficient and the minimum sum of squares of error E are obtained through continuous iteration, and the precise center coordinates are obtained through fitting.

$$E=\sum(\sqrt{(X_i-X_{\mathrm{B}})^2+(Y_i-Y_{\mathrm{B}})^2}-R)^2. \tag{2}$$

The spherical center coordinates can be obtained by using the fitting circle radius R_{B} and the known radius R_{A} of the target sphere, namely

$$X_{\mathrm{A}}=X_{\mathrm{B}},$$
$$Y_{\mathrm{A}}=Y_{\mathrm{B}},$$
$$Z_{\mathrm{A}}=-({R_{\mathrm{A}}}^2-{R_{\mathrm{B}}}^2)^{(1/2)}. \tag{3}$$

When the target ball rotates from the initial position and passes through the laser radar scanning plane, the fitting circle radius R_{B} becomes larger from small to large, and the maximum R_{B} is infinitely close to R_{A}. The scanning arc range is from the initial fitting R_{B} to the hemisphere smaller than R_{A}. Therefore, Z_{A} is negative in the laser radar coordinate system, and the data set $\boldsymbol{N}$ is obtained by collecting n times of ball center data within this range. The starting rotation position of the target ball in the rotary table coordinate system is the same as above. The position of the acquisition ball center is the n times position when the target ball passes through the laser radar scanning plane. Since the distance between the target ball center and the rotary table center is known, the position of the target ball center in the rotary table coordinate system relative to the origin O_1 can be calculated according to the rotary table rotation angle, which is expressed in (X_{C}, Y_{C}, Z_{C}), that is

$$X_{\mathrm{C}}=-S*\sin\theta,$$
$$Y_{\mathrm{C}}=\pm S*\cos\theta,$$
$$Z_{\mathrm{C}}=h, \tag{4}$$

where h is the height from the target ball center to the rotary table, and S is the distance from O_1 to the projection of the target ball center on the X_1Y_1 plane, and θ is the angle at which the target ball rotates from its starting position. The target ball rotates counterclockwise, so X_{C} is negative, and the positive or negative Y_{C} depends on the rotation angle. Less than 90° is positive, and more than 90° is negative. The data set $\boldsymbol{M}$ is obtained by collecting n times of spherical center data with the above method.

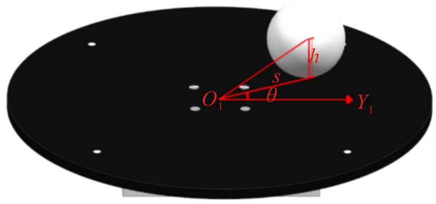


Fig.3 Solution relationship of target sphere coordinate information

The coordinate conversion parameters $\boldsymbol{R}$ and $\boldsymbol{T}$ are solved through the collected data sets $\boldsymbol{N}$ and $\boldsymbol{M}$. According to the previous n data collection, the point pair relationship between the two sets of data sets can be determined, and the conversion parameter values $\boldsymbol{R}$ and $\boldsymbol{T}$ corresponding to the minimum value of the conversion error function can be calculated.

The main steps are as follows.

First, the centroids corresponding to the points in the two sets of data sets are calculated respectively as

$$\boldsymbol{u}_{\boldsymbol{N}}=1/n\sum_{i=1}^{n}\|\boldsymbol{N}_i\|^2, \tag{5}$$
$$\boldsymbol{u}_{\boldsymbol{M}}=1/n\sum_{i=1}^{n}\|\boldsymbol{M}_i\|^2. \tag{6}$$

Convert the points in the two sets of data sets to the coordinate system with the corresponding centroid as the origin:

$$\boldsymbol{N}'=(\boldsymbol{N}_i-\boldsymbol{u}_{\boldsymbol{N}})=(\boldsymbol{N}_i'), \tag{7}$$
$$\boldsymbol{M}'=(\boldsymbol{M}_i-\boldsymbol{u}_{\boldsymbol{M}})=(\boldsymbol{M}_i'). \tag{8}$$

The coordinate conversion error functions of the two sets of data sets are as follows[11]

$$E(\boldsymbol{R},\boldsymbol{T})=1/n\sum_{i=1}^{n}\|\boldsymbol{M}_i-(\boldsymbol{R}\boldsymbol{N}_i+\boldsymbol{T})\|^2=$$
$$1/n\sum_{i=1}^{n}\|\boldsymbol{M}_i-(\boldsymbol{R}\boldsymbol{N}_i+\boldsymbol{T})-\boldsymbol{u}_{\boldsymbol{M}}+\boldsymbol{u}_{\boldsymbol{M}}+\boldsymbol{R}\boldsymbol{u}_{\boldsymbol{N}}-\boldsymbol{R}\boldsymbol{u}_{\boldsymbol{N}}\|^2. \tag{9}$$

After expanding the error function. Since the sum of cross terms is 0 and $\boldsymbol{T}$ can be derived from $\boldsymbol{R}$, the error function can be simplified as

$$E(\boldsymbol{R},\boldsymbol{T})=1/n\ \sum_{i=1}^{n}(-2{\boldsymbol{M}_i'}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{N}_i'), \tag{10}$$

of which

$$\sum_{i=1}^{n}{\boldsymbol{M}_i'}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{N}_i'=\sum_{i=1}^{n}\mathrm{trace}(\boldsymbol{R}\boldsymbol{M}_i'{\boldsymbol{N}_i'}^{\mathrm{T}})=\mathrm{trace}(\boldsymbol{R}\boldsymbol{H}),$$
$$\boldsymbol{H}=\sum_{i=1}^{n}\boldsymbol{M}_i'{\boldsymbol{N}_i'}^{\mathrm{T}}. \tag{11}$$

Calculate $\boldsymbol{W}$ as

$$\boldsymbol{W}=1/n\boldsymbol{H}=\boldsymbol{U}\begin{bmatrix}\delta_1 & 0 & 0\\ 0 & \delta_2 & 0\\ 0 & 0 & \delta_3\end{bmatrix}\boldsymbol{V}^{\mathrm{T}}. \tag{12}$$

The minimum value of the error function is the maximum value, when the maximum value is

$$\boldsymbol{R}=\boldsymbol{V}\boldsymbol{U}^{\mathrm{T}},$$
$$\boldsymbol{T}=\boldsymbol{u}_M-\boldsymbol{R}\boldsymbol{u}_N, \tag{13}$$

where $\boldsymbol{W}=\boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^{\mathrm{T}}$, $\boldsymbol{U}$ and $\boldsymbol{V}$ are orthogonal identity matrices, and $\boldsymbol{\Sigma}$ is a diagonal matrix.

Through the analysis of the above external parameter calibration methods of laser radar, it can be seen that the factors affecting the external parameter calibration accuracy mainly come from the fitting error caused by the spherical center fitting when obtaining the data set in the laser radar coordinate system. Therefore, this paper analyzes the accuracy of the above parts in the calibration process by setting a data set similar to the actual calibration process[12].

It can be seen from the above analysis that what the laser radar scans when scanning the target ball is an arc formed by an approximately uniformly distributed point on the surface of the target ball, so the set fitting data set is also an arc formed by an approximately uniformly distributed number of points. In this analysis, an arc curve with a center angle of 120° will be intercepted from a circle with a diameter of 8 cm, and the curve will be separated by a certain included angle. Then, the data set $\boldsymbol{N}$ in the laser radar coordinate system is simulated by distributing ±0.5 cm error points at each interval.

The circle is fitted by changing the included angle between points for many times, and the influence of the included angle on the accuracy of the fitted circle is analyzed by taking the difference between the fitted radius and the actual radius as the error, so as to select the optimal laser radar scanning resolution for data acquisition[13,14].

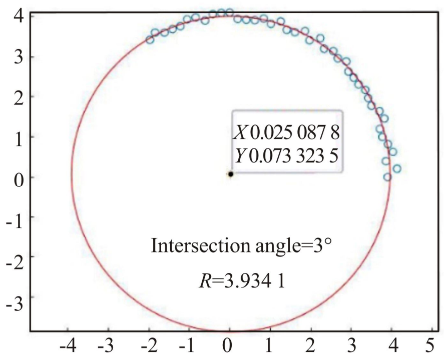


Fig.4 Data set under the simulated laser radar coordinate system

By analyzing the above data, it can be found that the fitting accuracy decreases linearly with the increase of the included angle, which affects the fitting accuracy of the center coordinates and radius. Therefore, according to the resolution options provided by the laser radar, it is the best to select 0.167° as the resolution of the data collected this time.

In order to verify the feasibility of the calibration method described in this paper, an experimental system is built. The laser in this system uses the LMS511-20100pro equipment of sick company, and the rotary table uses the DZY200RA200 servo driver to control the rotation of the rotary table to obtain the 3D point cloud information. This system is flexible, concise and economical, especially suitable for the rapid scanning and acquisition of 3D data of industrial parts. The experimental content is divided into three parts. The first part calculates the calibration parameters through the calibration method, the second part analyzes the error of the calibration results, and the third part uses the experimental system and the calculated calibration parameters to scan the 3D point cloud of the actual workpiece, and converts the setting information of the point cloud image to the rotary table coordinate system, and verifies the effectiveness of this method by comparing the workpiece size in the point cloud with the actual size.

Tab.1 Center coordinates and radius errors of the fitting circle under different included angles

Center angle (°)	Fitting radius (cm)	Fitting center coordinates (cm)	Radius error (cm)
3°	3.934 1	(0.025,0.073)	0.065 9
5°	3.898 9	(0.055,0.119)	0.101 1
8°	3.822 9	(0.138,0.217)	0.177 1
10°	3.795 3	(0.211,0.283)	0.204 7
12°	3.809 3	(0.033,0.158)	0.190 7
15°	3.742 1	(0.335,0.236)	0.257 9

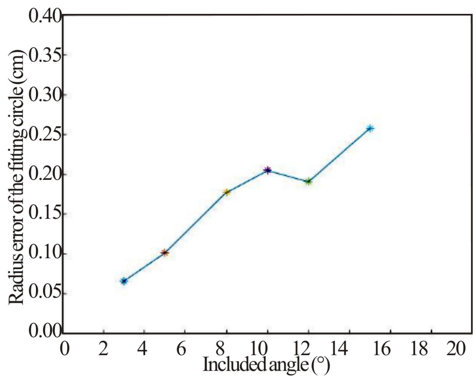


Fig.5 Radius errors of the fitting circle under different included angles

The target ball selected in this experiment is a ceramic standard ball with a diameter of 25.400 7 mm, which is fixed on the rotary table through the magnetic suction seat. The height h from the target ball center to the rotary table is 167.7 mm, the diameter of the rotary table is 50 cm, and the distance from the center of the bottom of the magnetic suction seat to the center of the rotary table is 12 cm. The laser radar scanner is fixed above the rotary table with a scanning resolution of 0.167°. The laser radar scanning system is shown in Fig.6.

Fig.6 Scanning system and target ball

Use the above method to scan the target ball and obtain 12 groups of data in two coordinate systems respectively. If the target ball is at the same height in the process of data collection, the accuracy of calibration parameter solution will be reduced. Therefore, in the process of data collection of each group, the height of the target ball will be increased by 10—30 mm at random. Because increasing the height of the target ball manually will cause errors, so the movement of the target ball will be converted to the relative movement of lidar. The cross is used to accurately adjust the height of the laser radar to replace the adjustment of the height of the target ball. The data collected under the laser radar coordinate system has useless noise that needs to be filtered. The filtered target ball arc data is fitted to obtain the spherical center coordinates. Finally, data sets $\boldsymbol{M}$ and $\boldsymbol{N}$ are obtained. As shown in Fig.7, the data in the red box is the target ball arc data obtained by scanning.

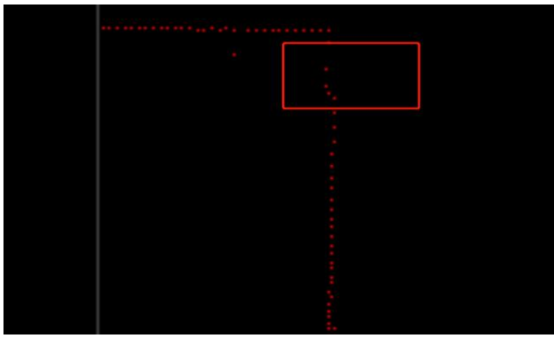


Fig.7 Target sphere arc obtained by scanning

By solving the coordinate conversion parameters $\boldsymbol{R}$ and $\boldsymbol{T}$ through the above algorithm, the conversion matrix is obtained as

$$\boldsymbol{Trans}=\begin{bmatrix} 99.4 & -64.0 & -993.0 & -160.1 \\ -968.6 & -234.7 & -81.8 & 242.7 \\ -227.8 & 970.0 & -85.3 & 386.6 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{14}$$

where the root mean square error (*RMSE*) is 0.004 295 close to 0, so the conversion matrix is correct.

In order to prove the validity of the calibration results, it is necessary to verify the accuracy of the solved conversion parameters. The collected data in the laser radar coordinate system is converted to the turntable coordinate system using the conversion parameters. The converted data set is represented by $\boldsymbol{P}$. The error between the converted data and the data in the actual turntable coordinate system is calculated, and the error is represented by the distance of the corresponding point pair in 3D space as[15]

$$D=\sqrt{(X_P-X_M)^2+(Y_P-Y_M)^2+(Z_P-Z_M)^2}. \tag{15}$$

Tab.2 Calibration result error (unit: mm)

Converted data			Actual data			Error
X_P	Y_P	Z_P	X_M	Y_M	Z_M	D
−115.112	33.794	167.606	−115.100	33.800	167.700	0.095 0
−115.758	31.491	177.566	−115.700	31.600	177.700	0.182 2
−116.322	29.196	182.602	−116.300	29.300	182.700	0.144 6
−117.017	27.057	185.786	−116.900	27.100	185.700	0.151 4
−117.540	24.991	189.736	−117.400	24.900	189.700	0.170 8
−117.796	22.643	181.676	−117.800	22.600	181.700	0.049 4
−118.170	20.206	193.734	−118.300	20.300	193.700	0.164 0
−118.667	17.919	186.680	−118.600	18.100	186.700	0.194 0
−118.808	15.865	194.734	−118.900	15.800	194.700	0.117 7
−119.173	13.394	187.707	−119.200	13.500	187.700	0.109 6
−119.437	11.157	182.631	−119.500	11.200	182.700	0.102 9
−119.532	8.843	191.691	−119.700	8.900	191.700	0.177 6

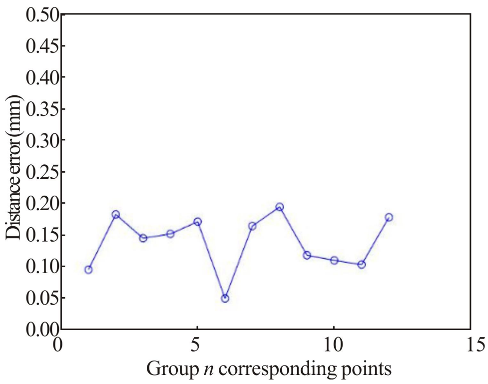


Fig.8 Calibration result error

By calculating the distance error D between 12 groups of corresponding point pairs, it can be seen that the maximum distance error between the converted data and the data under the actual rotary table coordinate system is 0.194 0 mm, the minimum error is 0.049 4 mm, the overall error remains within the accuracy range of 0.1 mm, and the calculated conversion matrix's $RMSE$=0.004 295 meets the accuracy requirements of the project.

The above conversion matrix is applied to the data acquisition of the actual workpiece, and the validity and versatility of the calibration parameters are verified by converting different data sets. The physical test is divided into two parts to verify the size of the carton and the sphericity error of the target ball respectively.

In the first part, the 3D scanning of the rectangular carton is carried out to obtain the data in the laser radar coordinate system, and the data of the carton in the turntable coordinate system is obtained through the ***Trans*** conversion. The left and right sides of Fig.9 are the point cloud data before and after the carton conversion.

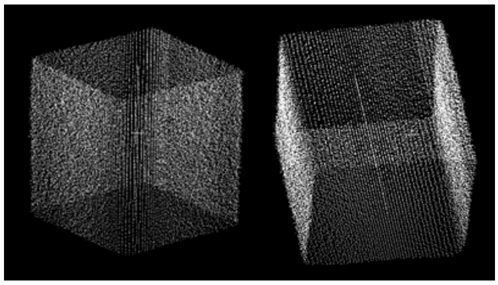
Compare the converted data with the actual measured data, and select the length, width and height of the carton as the comparison object in the comparison process. The size of the carton point cloud in the turntable coordinate system is measured by CloudCompare point cloud processing software[16].

It can be seen from the analysis data that the dimensional error of carton length, width and height is stable in the range of 0.1 mm, and the error accuracy meets the actual demand.

Tab.3 Carton size data (unit: mm)

	Actual size	Point cloud size	Error
Length	375.000	375.055	0.055
Wide	375.000	375.098	0.098
High	455.000	455.153	0.153

In the second part, the calibration ball is scanned in three dimensions, and the steps are the same as those of the above cartons. The point cloud data of the calibration ball under the turntable coordinate system is obtained. The point cloud data is directly filtered and denoised, and the calibration ball is fitted by using the least square method to obtain the diameter and spherical center coordinates of the fitting ball.



(a) (b)

Fig.9 Point cloud data (a) before and (b) after carton conversion

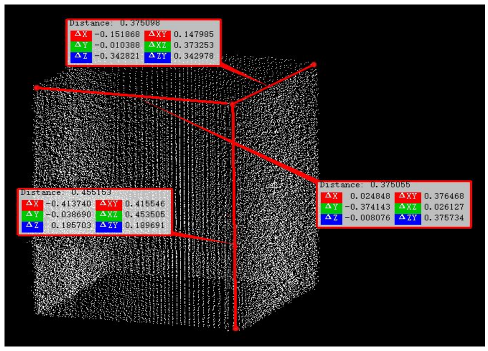


Fig.10 Point cloud size of carton under the rotary table coordinate system

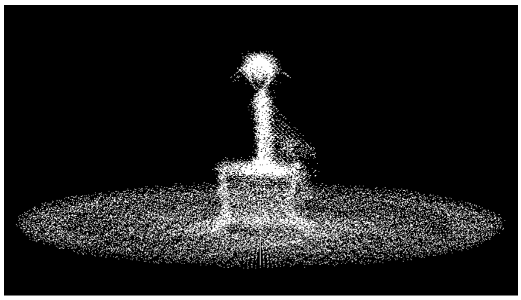
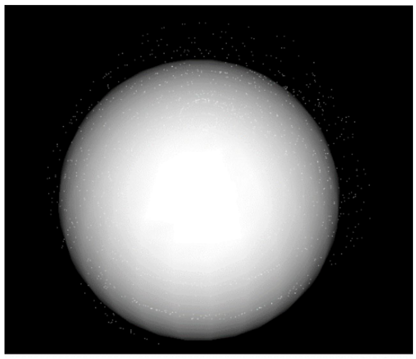


Fig.11 3D point cloud data of the calibration ball



Center(-0.005 271 38, 0.001 004 63,0.230 899)
Radius (0.012 735 9) *RMS*=0.002 482 28

Fig.12 Fitting spherical radius and spherical center coordinates

The standard diameter of the calibration ball is 25.400 7 mm, the diameter of the fitting ball is 25.471 8 mm, the error is 0.071 1 mm, and the root mean square is 0.002 482 28, which is within the error range of 0.1 mm, meeting the accuracy requirements.

In this paper, aiming at the problem of external parameter calibration of the rotary 3D data scanning method combined with lidar and turntable, a method is proposed. The target ball is used to collect the data sets under the lidar coordinate system and turntable coordinate system, and the coordinate conversion parameters of the two sets of data sets are solved. The experimental results show that the *RMSE* of the conversion parameters is 0.004 295, and the error between the size of the conversion point cloud object and the actual object size is stable within the accuracy range of 0.1 mm, which meets the actual requirements, proving that this method is effective and accurate.

Statements and Declarations

The authors declare that there are no conflicts of interest related to this article.

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