BER performance analysis of an OFDM-based visible light communication system using DHT post-coding*

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(Received 12 October 2021; Revised 23 February 2022)

The theoretical analysis of the bit error rate (*BER*) performance in a discrete Hartley transform (DHT) post-coded orthogonal frequency division multiplexing (OFDM) visible light communication (VLC) system over an additive white Gaussian noise (AWGN) channel is presented. The theoretical analysis results show that the proposed DHT post-coding scheme does not degrade the *BER* performance of the post-coded OFDM VLC systems. The analysis result is confirmed by our simulation results for random bit source. In addition, the peak-to-average power ratio (*PAPR*) of the DHT post-coded OFDM is evaluated by simulation. The simulation results show that DHT post-coding can greatly reduce of the *PAPR* of the optical OFDM system.

Document code: A **Article ID:** 1673-1905(2022)05-0288-5

DOI https://doi.org/10.1007/s11801-022-1160-4

Visible light communication (VLC), which is a promising complementary technique to the conventional radio frequency (RF) communication, has attracted attention of researchers[1,2]. Orthogonal frequency division multiplexing (OFDM) technique has been applied in VLC in order to improve the spectral efficiency of VLC systems. In OFDM-based VLC, intensity modulation with direct detection (IM/DD) is commonly deployed. Thus, the transmitted signal of OFDM-based VLC has to be real-valued and positive. direct current-biased optical OFDM (DCO-OFDM) is one of the popular modulation schemes. In DCO-OFDM systems, the frequency domain signals need to satisfy Hermitian symmetry in order to generate real-valued OFDM signal. Then, the resulting OFDM signal is converted into a non-negative signal by adding a direct current (DC) bias. In our study, only DCO-OFDM-based VLC will be considered due to its superior spectral efficiency. For DCO-OFDM VLC systems, the high peak-to-average power ratio (*PAPR*) of a real-valued OFDM signal is the main issue that could cause serious performance degradation in the presence of nonlinear characteristic of a light emitting diode (LED). Among various *PAPR* reduction techniques, precoding method is considered as a potential approach, as it can reduce *PAPR* without degrading bit error rate (*BER*) performance of an OFDM system.

Currently, some pre-coders, such as discrete Fourier transform (DFT), discrete Hartley transform (DHT) and discrete cosine transform (DCT) precoding schemes, are also employed in optical OFDM systems[3,4]. However, the large number of studies on *PAPR* reduction using pre-coding are based on simulation and experimental approaches. There are some reports on the theoretical analysis on *BER* of precoded RF OFDM systems. In Ref.[5], the authors studied the *BER* performance of the precoded OFDM over additive white Gaussian noise (AWGN) channel by theoretical analysis approach. The theoretical analysis results show that the *BER* of the precoded OFDM is the same as that of the original OFDM over AWGN channel. In Ref.[6], the theoretical analysis and simulation results both show that the Wash-Hadamard precoded OFDM and the original OFDM systems over an AWGN channel are identical.

Recently, post-coding has attracted attention for reducing *PAPR* of OFDM signals. In Ref.[7], a new *PAPR* reduction technique, namely, Zadoff-Chu matrix transform (ZCT) post-coding was proposed in RF OFDM systems. In Ref.[8], the authors discussed and analyzed the performance of post-coded RF OFDM systems. In Ref.[9], the simulation results show that DHT post-coding does not degrade the *BER* performance in DC-biased optical OFDM systems with great *PAPR* reduction. In Ref.[10], the authors proposed a novel post-coding scheme based on Gaussian orthogonal matrix in OFDM VLC systems to reduce *PAPR* without degrading *BER* performance. However, the above mentioned previous works usually employed the simulation and experiment methods to study the performance of post-coded OFDM VLC systems. The theoretical analysis of the *BER* performance of a post-coded OFDM VLC

* This work has been supported in part by the Key Laboratory of Universal Wireless Communications (BUPT), Ministry of Education, China (No.KFKT-2020103), and the Zhejiang Provincial Key Natural Science Foundation of China (No.LZ21F010001).

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system has not been studied.

Inspired by the theoretical analysis methods in Refs.[5,6] for RF OFDM systems, we will address the impact of post-coding on the *BER* performance of a DCO-OFDM-based VLC system by theoretical analysis approach. We mainly study the *BER* performance of DHT post-coded OFDM-based VLC over an AWGN channel. The VLC channel is usually considered to be a typical line-of-sight channel with AWGN[11,12]. According to the principle of OFDM, a frequency selective channel can be divided into many sub-channels, and each sub-channel is regard as approximately flat channels. Thus, the performance of OFDM systems over an AWGN channel forms a theoretical basis, from which the results of OFDM systems over a frequency selective channel can be obtained[13].

In this work, theoretical analysis of the DHT post-coded DCO-OFDM system over an AWGN channel is presented. Our theoretical analysis results prove that the post-coding does not degrade *BER* performance of the DHT post-coded OFDM. This point has been verified by our simulation experiments. So DHT post-coding can greatly reduce the *PAPR* of OFDM signals compared to the conventional DHT pre-coding and the original OFDM and meantime maintain the same *BER* performance.

Let us consider an OFDM VLC system with N subcarriers. Let input data vector $\boldsymbol{X}=\{X_k, k=0, 1,..., N-1\}$ denote the input of an inverse fast Fourier transform (IFFT) operation. To produce a real-valued OFDM signal, the elements of $\boldsymbol{X}$ must satisfy $X_k = X_{N-k}^*$. After performing an N-point IFFT on $\boldsymbol{X}$, the produced real-valued signal can be written as follows

$$x_n = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} X_k \exp(\frac{\mathrm{j}2\pi kn}{N}), \quad 0 \le n \le N-1 . \qquad (1)$$

The *PAPR* of OFDM signal, which is defined as the ratio between the maximum peak power and the average power of the transmitted OFDM signals, is commonly defined as

$$PAPR = \frac{\max\limits_{0\le n\le N-1}\left[\left|x_n\right|^2\right]}{E\left\{\left|x_n\right|^2\right\}} = \frac{\left\|x_n\right\|_\infty^2}{P_x} = \frac{\left\|x_n\right\|_\infty^2}{\left\|x_n\right\|_2^2/N}, \qquad (2)$$

where $\|\cdot\|_2$ is the 2-norm and $\|\cdot\|_\infty$ stands for the ∞-norm, and $E\{\cdot\}$ denotes the mathematical expectation. The *PAPR* performance of an OFDM system can be evaluated using the complementary cumulative distribution function (*CCDF*). The *CCDF* of *PAPR* (namely, P_c) can be expressed as

$$P_c=\Pr\{PAPR>\zeta_P\}, \qquad (3)$$

where P_c indicates the probability that *PAPR* exceeds a particular value ζ_P.

Different from the conventional precoding, which is used before the IFFT unit, the post-coding is used after IFFT unit in post-coded OFDM systems. Fig.1 shows the block diagram of the DHT post-coded OFDM-based VLC system. A DHT matrix with size of $N\times N$ is employed after IFFT unit to reduce the *PAPR* of OFDM signals. In the proposed DHT post-coded OFDM system, the IFFT transforms a complex baseband modulated signal vector $\boldsymbol{X}$ into a new vector $\boldsymbol{x}$ with size N, which is written as

$$\boldsymbol{x}=\boldsymbol{W}^{\mathrm{H}}\boldsymbol{X}, \qquad (4)$$

where $\boldsymbol{W}^{\mathrm{H}}$ is an $N\times N$ DFT matrix with its (n, k)th element of $(1/\sqrt{N})\mathrm{e}^{-\mathrm{j}2\pi kn/N}$, and $(\cdot)^{\mathrm{H}}$ denotes the Hermitian transpose operator. Then the DHT post-coding matrix converts the time-domain signal into the new transform-domain signal, which can be expressed as

$$\boldsymbol{z}=\boldsymbol{F}\boldsymbol{X}, \qquad (5)$$

where $\boldsymbol{F}$ is an $N\times N$ DFT matrix, and its (p, l)th element can be expressed as

$$F_{q,l} = \mathrm{cas}(\frac{2\pi ql}{N}) = \cos(\frac{2\pi ql}{N}) + \sin(\frac{2\pi ql}{N}). \qquad (6)$$

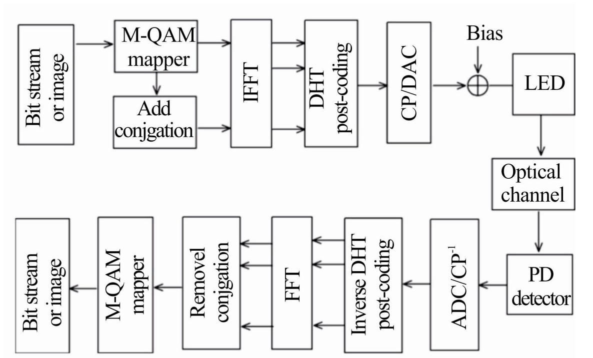

Fig.1 Block diagram of an OFDM-based VLC system with DHT post-coding

Substituting Eq.(4) into Eq.(5), the signal $\boldsymbol{z}$ can be rewritten as

$$\boldsymbol{z}=\boldsymbol{F}\boldsymbol{X}=\boldsymbol{F}\boldsymbol{W}\boldsymbol{X}, \qquad (7)$$

where the discrete OFDM signal $\boldsymbol{z}$ is converted into continuous OFDM signal $\boldsymbol{z}(t)$ by digital to analog converter (DAC) module. Then the bipolar signal $\boldsymbol{z}(t)$ is added a DC bias B_{DC} in order to be converted into a unipolar signal $\boldsymbol{z}_{\mathrm{DC}}(t)$ that drives the transmit LED. The biased signal $\boldsymbol{z}_{\mathrm{DC}}(t)$ is given by

$$\boldsymbol{z}_{\mathrm{DC}}(t)= \boldsymbol{z}(t) + B_{\mathrm{DC}}, \qquad (8)$$

where $B_{\mathrm{DC}} = k\sqrt{E\{z^2(t)\}}$ denotes DC bias level. And k is a proportionality constant. At the receiver, the received signal is first converted into an electrical signal using a photo-diode. The electrical signal can be written as[14]

$$\boldsymbol{r}(t)=\eta\cdot \boldsymbol{z}(t)+\boldsymbol{n}(t), \qquad (9)$$

where η is the responsivity of the photo-electrical converter in A/W. To simplify and not impact the theoretical analysis, we set $\eta=1$[14]. $\boldsymbol{n}(t)$ is a zero-mean Gaussian process with the variance of σ^2[14]. Then the received discrete electronic signal $\boldsymbol{r}$ can be written as

$$\boldsymbol{r}=\boldsymbol{z}+\boldsymbol{n}. \tag{10}$$

The inverse post-coded transform $\boldsymbol{F}^{\mathrm{H}}$ is applied to the signal vector $\boldsymbol{r}$, and a new transformed signal is given as

$$\hat{\boldsymbol{z}} = \boldsymbol{F}^{\mathrm{H}}\boldsymbol{r} = \boldsymbol{F}^{\mathrm{H}}(\boldsymbol{z}+\boldsymbol{n}) = \boldsymbol{F}^{\mathrm{H}}\boldsymbol{z} + \boldsymbol{F}^{\mathrm{H}}\boldsymbol{n} = \boldsymbol{F}^{\mathrm{H}}\boldsymbol{z} + \boldsymbol{u}. \tag{11}$$

Finally, after $\hat{\boldsymbol{z}}$ is processed by the FFT unit, the output of the FFT can be given as

$$\begin{aligned}\hat{\boldsymbol{X}} &= \boldsymbol{W}\hat{\boldsymbol{z}} = \\ &\boldsymbol{W}\left(\boldsymbol{F}^{\mathrm{H}}\boldsymbol{z} + \boldsymbol{F}^{\mathrm{H}}\boldsymbol{n}\right) = \\ &\boldsymbol{W}\boldsymbol{F}^{\mathrm{H}}\left(\boldsymbol{F}\boldsymbol{W}^{\mathrm{H}}\boldsymbol{X}\right) + \boldsymbol{W}\boldsymbol{F}^{\mathrm{H}}\boldsymbol{n} = \\ &\boldsymbol{X} + \boldsymbol{W}\boldsymbol{u} = \\ &\boldsymbol{X} + \varsigma,\end{aligned} \tag{12}$$

where $\boldsymbol{u}$ is defined as $\boldsymbol{u}=\boldsymbol{F}^{\mathrm{H}}\boldsymbol{n}$, and $\varsigma=\boldsymbol{W}\boldsymbol{u}$. The kth element of $\hat{\boldsymbol{X}}$ can be expressed as

$$\hat{X}_k = X_k + W_k u = X_k + \varsigma_k. \tag{13}$$

Based on the post-coded OFDM signal model of Eq.(13), we can analyze the variance of the noise ς_k in post-coded OFDM. Furthermore, we can analyze the *BER* performance of the post-coded OFDM VLC over AWGN channel. For the conventional multiple quadrature amplitude modulation (M-QAM) system over the AWGN channel, the theoretical *BER* formula is given as[15]

$$P_{\mathrm{b,MAQM}} = \left(\frac{4-2^{(2-m/2)}}{m}\right) Q\left(\sqrt{\frac{3\gamma_0}{(M-1)}}\right), \tag{14}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} \mathrm{e}^{\frac{-t^2}{2}}\mathrm{d}t$ denotes the Q function, and $m=\log_2 M$ is the number of bits per constellation point. γ_0 is the signal-to-noise ratio (*SNR*) at the receiver, which is defined as

$$\gamma_0 = \sigma_X^2 / \sigma^2, \tag{15}$$

where σ_X^2 and σ^2 denote the desired signal average power and the Gaussian noise average power, respectively.

Similar to the analysis in the pre-coded OFDM[5,6], we can obtain the noise variance in post-coded OFDM based on Eq.(13). Let each element in ς be expressed as $\varsigma_k = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} u_n W_{k,n}$. Thus, the variance of ς_k can be given as

$$\begin{aligned}E\left\{|\varsigma_k|^2\right\} &= E\left\{\varsigma_k\varsigma_k^*\right\} = \frac{1}{N^2}\sum_{k=0}^{N-1}\left(\sum_{n=0}^{N-1} u_n W_{k,n}\sum_{m=0}^{N-1} u_m^* W_{k,m}^*\right) = \\ &\frac{1}{N^2}\sum_{k=0}^{N-1}\left(\sum_{n=0}^{N-1}|u_n|^2 |W_{k,n}|^2\right) + \\ &\frac{1}{N^2}\sum_{k=0}^{N-1}\left(\sum_{n=0}^{N-1}\sum_{m=0,n\neq m}^{N-1} u_n u_m^* W_{k,n} W_{k,m}^*\right).\end{aligned} \tag{16}$$

Due to the orthogonality of the basis function of the DFT matrix $\boldsymbol{W}$, the second term of the right-hand side of Eq.(16) is equal to 0.

$$\begin{aligned}E\left\{|\varsigma_k|^2\right\} &= \frac{1}{N^2}\sum_{k=0}^{N-1}\sum_{n=0}^{N-1}|u_n|^2 |W_{k,n}|^2 = \\ &\frac{1}{N^2}\sum_{n=0}^{N-1}|u_n|^2\sum_{k=0}^{N-1}|W_{k,n}|^2 = \\ &\frac{1}{N}\sum_{n=0}^{N-1}|u_n|^2 = E\{|u_n|^2\}.\end{aligned} \tag{17}$$

Furthermore, due to $\boldsymbol{u}=\boldsymbol{F}^{\mathrm{H}}\boldsymbol{n}$, the variance of $\boldsymbol{u}$ can be written as

$$\begin{aligned}E\left\{|u_l|^2\right\} &= E\left\{u_l u_l^*\right\} = \\ &\frac{1}{N^2}\sum_{l=0}^{N-1}\left(\sum_{q=0}^{N-1} n_q^* F_{l,q}^* \sum_{p=0}^{N-1} n_p F_{l,p}\right) = \\ &\frac{1}{N^2}\sum_{l=0}^{N-1}\left(\sum_{q=0}^{N-1}|n_q|^2 |F_{l,q}|^2\right) + \\ &\frac{1}{N^2}\sum_{l=0}^{N-1}\left(\sum_{q=0}^{N-1}\sum_{p=0,q\neq p}^{N-1} n_q^* u_p F_{l,q}^* F_{l,p}\right).\end{aligned} \tag{18}$$

Due to the orthogonality of the matrix $\boldsymbol{F}^{\mathrm{H}}$, the second term on the right-hand side of Eq.(18) is equal to 0. Thus, Eq.(18) can be rewritten as

$$\begin{aligned}E\left\{|u_l|^2\right\} &= \frac{1}{N^2}\sum_{l=0}^{N-1}\left(\sum_{q=0}^{N-1}|n_q|^2 |F_{l,q}|^2\right) = \\ &\frac{1}{N^2}\sum_{q=0}^{N-1}|n_q|^2\sum_{l=0}^{N-1}|F_{l,q}|^2 = \\ &\frac{1}{N}\sum_{q=0}^{N-1}|n_q|^2 = E\{|n_q|^2\} = \sigma^2.\end{aligned} \tag{19}$$

Based on Eq.(19), the noise variance ς can be expressed as

$$E\{|\varsigma_k|^2\} = \sigma^2. \tag{20}$$

Based on the above results, it can be seen that post-coding does not change the noise variance when compared with that of the original OFDM system. On the other hand, based on the signal model of Eq.(13), it can be seen that after post-coding operation, the signal average power of the transmitted signal $\boldsymbol{X}$ has not been changed.

According to Eq.(13), the *SNR* values of the post-coded OFDM-VLC system can be calculated as

$$\gamma_{\mathrm{post}} = \sigma_X^2 / \sigma^2 = \gamma_0. \tag{21}$$

It is clear from Eq.(21) that the *SNR* value of the post-coded OFDM system over an AWGN channel is the same as that of the original OFDM system in Eq.(15).

Thus, according to the *BER* formula of Eq.(14), the proposed DHT post-coded system has precisely the same *BER* performance as the original OFDM system.

In the following section, the *PAPR* performance of the proposed scheme is first evaluated by simulation experiments. In the simulation, the number of sub-carriers is 256. Fig.2 shows the *CCDF* comparison of *PAPR* of the DHT post-coded DCO-OFDM, DHT pre-coded DCO-OFDM, and original DCO-OFDM for quadrature phase shift keying (QPSK) modulation. At $CCDF=10^{-3}$, DHT post-coded OFDM scheme can reduce the *PAPR* by approximately 9.3 dB and 8 dB compared with the original OFDM and the DHT pre-coded OFDM, respectively. From Fig.2, we can see that the proposed DHT post-coded OFDM can significantly reduce the *PAPR* of OFDM.

We now compare original DCO-OFDM, DHT pre-coded DCO-OFDM, and DHT post-coded DCO-OFDM signals in terms of the *BER* performance over an AWGN channel. The relationship between *BER* with *SNR*, which is defined as bit electrical energy normalized to the noise power spectral density ($E_{b(elec)}/N_0$), is shown in Fig.3. From Fig.3, it can be seen that the theoretical *BER* is the same as that of the post-coded OFDM for the bipolar QPSK OFDM signals. It is also seen that the *BER* performance of the post-coded OFDM is almost the same as that of the DHT pre-coded OFDM and original OFDM systems for the bipolar QPSK OFDM signals. For a DCO-OFDM system, the bias value is fixed at 7 dB, which is defined as $10\log_{10}(k^2+1)$ dB[13]. For the 7 dB bias case, the *BER* performance of the post-coded OFDM is lightly better than that of the precoded OFDM and original OFDM at higher *SNR*. This is because the clipping noise in post-coded OFDM is smaller than that of the precoded OFDM due to the better *PAPR* performance of the post-coded OFDM. The simulation results in Fig.3 show that the *BER* performance of the post-coded OFDM is in good agreement with the *BER* theoretical analysis above and the reported results in Ref.[9].

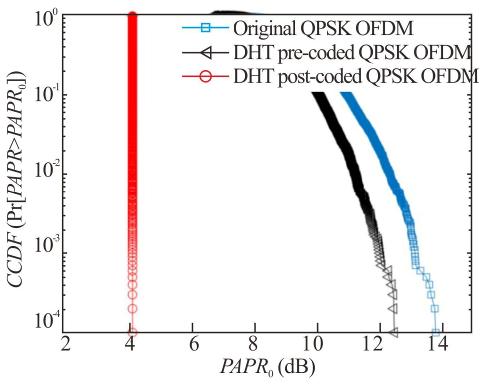

Fig.2 Comparison of *PAPR* reduction performance with various schemes

Fig.4 shows the performance comparison of the original OFDM, the precoded OFDM, and the post-coded OFDM over an AWGN channel for House image with size of 256×256. For the bipolar OFDM signal case, the *BER* performance of the precoded OFDM and post-coded OFDM is deteriorated lightly than that of the original OFDM signal for the image source. Furthermore, from Fig.4, we can see that the theoretical *BER* curve is also basically agreement with the simulation curves. Similar to Fig.3, for the 7 dB bias case, the *BER* performance of the post-coded OFDM is lightly better than that of the precoded OFDM and original OFDM at higher *SNR* case.

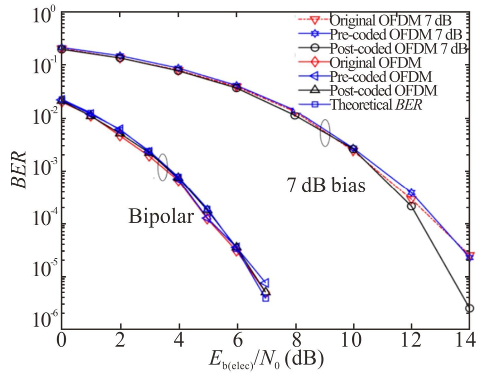

Fig.3 Comparison of *BER* performance with various schemes for random bit source

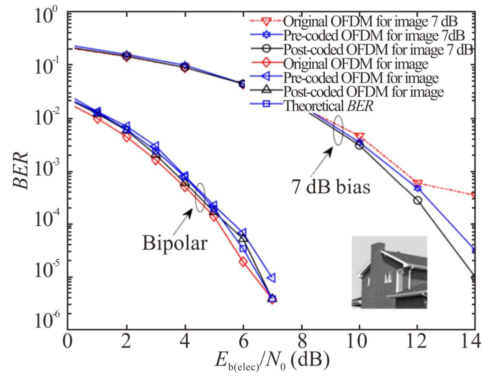

Fig.4 Comparison of *BER* performance with various schemes for House image

In this paper, a *BER* performance analysis of a DHT post-coed OFDM VLC system is studied over an ANGN channel. A closed-form expression for *BER* is derived for the proposed post-coded OFDM system. The theoretical analysis results state that post-coding does not change the *BER* performance of the proposed systems. The simulation results for *BER* also show that the *BER* performance of the proposed scheme is almost the same as that of the original OFDM system and the conventional DHT precoded OFDM system. The simulation results for *BER* confirm the theoretical analysis results. Furthermore, post-coding can greatly reduce the *PAPR* of OFDM signals. This can improve power efficiency of post-coded OFDM systems.

Statements and Declarations

The authors declare that there are no conflicts of interest related to this article.

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